Authors

Jeffrey Packer, Ph.D., D.Sc., P.Eng., is a Bahen/Tanenbaum Professor in Civil Engineering at the University of Toronto. He has a bachelor’s degree from the University of Adelaide, Australia (1972), a master’s degree from the University of Manchester, U.K. (1975), and doctorates from the University of Nottingham, U.K. (1978, 2006). His research and publications have concentrated on the behavior and design of hollow structural section (HSS) connections.

Donald Sherman, Ph.D., P.E., is professor emeritus at the University of Wisconsin–Milwaukee. He has bachelor’s and master’s degrees from Case Institute of Technology (1957, 1960) and a Ph.D. from the University of Illinois (1964). His research and publications include HSS member behavior and simple connections to HSS columns.

Maura Lecce, Ph.D. is a postdoctoral fellow at the University of Toronto. She has bachelor’s and master’s degrees from the University of Toronto (2000, 2001) and a Ph.D. from the University of Sydney, Australia (2006). Her research and publications include HSS connections and stainless steel light gage members.

Preface

This Design Guide is a supplement to the 13th edition of the American Institute of Steel Construction (AISC) Steel Construction Manual and its companion CD. The Manual contains sections on bolting to hollow structural sections (HSS), welding considerations for HSS, simple shear connections to HSS columns, fully restrained moment connections to HSS columns, and design considerations for HSS-to-HSS truss connections. The companion CD has seven examples of simple shear connections to HSS columns. Therefore, this Guide does not have a chapter on simple shear connections. The CD also has examples of a transverse plate on a rectangular HSS, a longitudinal plate on a round HSS, and HSS braces with end gusset plates, as well as examples of the design of cap plates, base plates and end plates on HSS members.

The examples in this Guide conform to the 2005 AISC Specification for Structural Steel Buildings. Both load and resistance factor design (LRFD) and allowable strength design (ASD) solutions are presented. References are given to applicable sections of the Specification and to design tables in the Manual. This Guide contains a few additional tables that are applicable to HSS connections. It is recommended that readers of this Guide first become familiar with the Specification provisions for HSS connections and the accompanying Specification Commentary.

Some of the material in this Guide is based on the AISC Hollow Structural Sections Connections Manual published in 1997. However, because the AISC Specification has evolved from that in effect in 1997, the 13th edition Manual and this Guide supersede the previous HSS Manual.

Chapter K of the Specification presents the criteria for forces (axial force, in-plane moment and out-of-plane moment) in branch members framing into a main member. In this Guide, these same equations appear in a tabular format with drawings showing the connection configuration. This format is easier to follow than the descriptive text in the Specification. The design examples of direct HSS-to-HSS connections refer to both the appropriate tables in this Guide and the Specification equations.
Table of Contents

CHAPTER 1 INTRODUCTION ................................................. 1
1.1 HSS AND BOX-SHAPED MEMBERS ............................... 1
1.2 HSS CONNECTION DESIGN STANDARDS AND SCOPE ................... 3
1.3 ADVANTAGES OF HSS .............................................. 3
1.4 OTHER CONSIDERATIONS ......................................... 4
1.4.1 Notch Toughness ............................................. 4
1.4.2 Galvanizing Issues .......................................... 4
1.4.3 Internal Corrosion .......................................... 4

CHAPTER 2 WELDING ...................................................... 5
2.1 TYPES OF HSS WELDS ......................................... 5
2.1.1 Fillet Welds .................................................. 5
2.1.2 PJP and CJP Groove Welds .............................. 6
2.1.3 Flare-Bevel and Flare-V Groove Welds ........................ 6
2.2 WELD INSPECTION ............................................. 7
2.3 EFFECTIVE SIZE OF FILLET WELDS ...................... 8
2.4 EFFECTIVE WELD LENGTH ................................... 8
2.5 WELDED JOINT DESIGN EXAMPLES ....................... 10

CHAPTER 3 MECHANICAL FASTENERS .................................. 15
3.1 FASTENERS IN SHEAR ......................................... 15
3.2 FASTENERS IN TENSION ...................................... 15
3.3 BOLTED JOINT DESIGN EXAMPLES ....................... 16

CHAPTER 4 MOMENT CONNECTIONS ................................. 29
4.1 W-BEAMS TO HSS COLUMNS ................................ 29
4.2 CONTINUOUS BEAM OVER HSS COLUMN ................. 29
4.3 THROUGH-PLATE CONNECTIONS ......................... 30
4.4 DIRECTLY WELDED CONNECTIONS ...................... 30
4.5 CONNECTION DESIGN EXAMPLES ....................... 31

CHAPTER 5 TENSION AND COMPRESSION CONNECTIONS ................. 49
5.1 TYPES OF END CONNECTIONS ............................... 49
5.2 END TEE CONNECTIONS ..................................... 50
5.3 SLOTTED HSS/GUSSET CONNECTION ................... 51
5.4 END PLATE ON ROUND HSS .................................. 51
5.5 END PLATE ON RECTANGULAR HSS WITH BOLTS ON TWO SIDES ............... 52
5.6 END PLATE ON RECTANGULAR HSS WITH BOLTS ON FOUR SIDES ............. 53
5.7 CONNECTION DESIGN EXAMPLES ....................... 55

CHAPTER 6 BRANCH LOADS ON HSS—AN INTRODUCTION ............. 73
6.1 PRINCIPAL LIMIT STATES ........................................ 73
6.1.1 Chord or Column Wall Plastification ................... 73
6.1.2 Chord Shear Yielding (Punching Shear) .................. 73
6.1.3 Local Yielding Due to Uneven Load Distribution ...................... 74
6.1.4 Chord or Column Sidewall Failure ...................... 75
6.2 DESIGN TIPS .................................................... 76

CHAPTER 7 LINE LOADS AND CONCENTRATED FORCES ON HSS ........ 77
7.1 SCOPE AND BASIS ............................................. 77
7.2 LIMIT STATES .................................................... 77
7.3 CONNECTION NOMINAL STRENGTH TABLES ..................... 79
7.4 LONGITUDINAL-PLATE AND CAP-PLATE CONNECTIONS .................. 83
7.5 CONNECTION DESIGN EXAMPLES ....................... 84

CHAPTER 8 HSS-TO-HSS TRUSS CONNECTIONS ........................ 91
8.1 SCOPE AND BASIS ............................................ 91
8.2 NOTATION AND LIMIT STATES ............................... 91
8.3 CONNECTION CLASSIFICATION ............................... 92
8.4 TRUSS MODELING AND MEMBER DESIGN ..................... 93
8.5 CONNECTION NOMINAL STRENGTH TABLES ..................... 96
8.6 CONNECTION DESIGN EXAMPLES ....................... 100

CHAPTER 9 HSS-TO-HSS MOMENT CONNECTIONS .................. 123
9.1 SCOPE AND BASIS ............................................ 123
9.2 NOTATION AND LIMIT STATES ............................... 123
9.3 CONNECTION NOMINAL CAPACITY TABLES ..................... 125
9.4 CONNECTION DESIGN EXAMPLES ....................... 128

SYMBOLS ........................................................................ 141
REFERENCES ................................................................... 144
Chapter 1
Introduction

In recent years, the popularity of hollow structural sections (HSS) has increased dramatically. The pleasing aesthetic appearance generated by architecturally exposed hollow sections is much favored by architects, and HSS also can provide reduced weight and surface area when compared to equivalent open sections. Some stunning examples of exposed HSS in building interiors are shown in Figure 1-1.

Connections usually have been the challenging aspect for the designer of structures that involve HSS. This AISC Design Guide demonstrates design methods for a wide range of connection types. Note that, in many cases, the local strength of the HSS at the connection is an integral part—and perhaps a limitation—of the design. Moreover, note that reinforcing the connections of HSS assemblies often is not an available option, for either architectural or fabrication reasons.

1.1 HSS AND BOX-SHAPED MEMBERS

HSS manufactured according to American Society for Testing and Materials (ASTM) standard A500 (ASTM, 2007a) are cold-formed in tube mills, and have an electric resistance welded (ERW) continuous seam weld. This “weld” is produced without the addition of any additional consumable. The weld bead on the outside is always removed, but the weld bead that results on the inside of the HSS is generally left in place. However, this inside weld bead can be removed at the point of manufacture if this requirement is specified to the tube mill; this may be desirable if one HSS is inserted into another, for example with telescoping poles.

Round, square and rectangular HSS produced in accordance with the ASTM A500 Grade B standard are readily available throughout North America. Rectangular HSS are also frequently termed “shaped” sections. ASTM A500 Grade C is increasingly available, and many HSS products are now dual-certified as meeting the requirements in both ASTM A500 Grade B and Grade C. The relative material strengths of these ASTM A500 HSS are shown in Table 1-1. Note that a particular grade has different yield strengths for round versus rectangular shapes.

In some parts of the United States, various pipe products are readily available and used in lieu of round HSS. ASTM A53 (ASTM, 2007c) Grade B pipe, which is included in the American Institute of Steel Construction (AISC) Specification for Structural Steel Buildings, hereafter referred to as the AISC Specification (AISC, 2005a), has a lower yield strength than its ASTM A500 counterpart (see Table 1-1). All load tables in the 13th edition AISC Steel Construction Manual, hereafter referred to as the AISC Manual (AISC, 2005b), for HSS are based on ASTM A500 Grade B strengths, and the load tables for pipe use ASTM A53 Grade B strengths.

Other North American HSS products that have properties and characteristics similar to the approved ASTM products are produced in Canada (CSA, 2004). This standard allows for two types of finished product: Class C (cold-formed) and Class H (cold-formed and stress relieved by heat treatment). Class H HSS have reduced levels of residual stress, which enhances their performance as compression members and may provide better ductility in the corners of rectangular HSS.

(a) Opryland Hotel, Nashville, Tennessee.

(b) Interior of Rock and Roll Hall of Fame, Cleveland, Ohio.

Fig. 1-1. Contemporary examples of HSS in construction.
It should also be noted that very large diameter tubular sections are available to American Petroleum Institute (API) specifications; many different grades are available, and specified outside diameters range from 0.405 in. to 84 in. (API, 2007).

The designation used to identify square and rectangular HSS is, for example:

**HSS8×4×¼**

In this designation, the whole numbers are the height and width, and the fraction is the nominal thickness. Decimal numbers are used for the outside diameter and nominal thickness in the designation of round HSS, for example:

**HSS6.000×0.375**

The designation for pipe is a traditional form for three grades, including:

- Standard; Std.
- Extra Strong; x-strong
- Double Extra Strong; xx-strong

The diameter designated for pipe is a nominal value between the specified inside and outside diameters, for example:

**Pipe 8 x-strong**

The dimensions and geometric properties of HSS and pipe are included in Part 1 of the AISC Manual in the following tables:

- Table 1-11: Rectangular HSS
- Table 1-12: Square HSS
- Table 1-13: Round HSS
- Table 1-14: Pipe

Dimensional tolerances of the products are also included in the following tables:

- Table 1-27: Rectangular and Square HSS
- Table 1-28: Round HSS and Pipe

The ASTM cold-formed material standard tolerances permit the wall thickness to be 10% under the nominal wall thickness. Consequently, the mills consistently produce HSS with wall thicknesses less than the nominal wall thickness. Section B3.12 in the AISC Specification accounts for this by designating a design wall thickness of 0.93 times the nominal thickness. The design wall thickness is included in the tables of dimensions and properties in the AISC Manual, and all properties (A, D/t, I, Z, S, etc.) are based on the design wall thickness.

Round, square and rectangular HSS manufactured according to the ASTM A500 standard are available in perimeters up to 64 in. and in thicknesses up to ¼ in. Larger sizes in square and rectangular sections are classified as box-shaped members in the AISC Specification. A standard product line of these box sections up to 128-in. perimeter is produced by placing two flat strips in a brake press to form two identical halves of a finished tube size. A backing bar is tack welded to each leg of one of the half sections. Then, the two half sections are fitted together toe-to-toe and submerged arc welded together to complete the square or rectangular section. These sections are produced with the full nominal thickness so that the design wall thickness of 0.93t does not apply.

The standard sizes of HSS and larger box-shaped members produced appear in HSS availability listings on the AISC website (www.aisc.org) and periodically in AISC’s Modern Steel Construction magazine. Tables of dimensions and section properties for larger box-shaped members can be obtained from the manufacturer.
The ASTM A501 standard (ASTM, 2007b) for hot-formed tubing is included in the AISC Specification even though these products have not been produced in North America for several decades. However, ASTM A501 has recently been revised to add Grade B, which is a hot-finished product with the mechanical properties shown in Table 1-1. The manufacturing process is similar to cold-formed HSS, but the final shaping and sizing are completed after the steel has been heated to a full normalizing temperature. These products are made by European manufacturers in round, square, rectangular, and elliptical shapes (Packer, 2008). Such sections are essentially produced to the European standard EN10210 Parts 1 and 2 (CEN, 2006a, 2006b), and the elliptical hollow sections have a major-to-minor axis dimension ratio on the order of 2:1.

Sections up to 16-in. square and 0.625-in. thick are produced with ERW seams and are available in several sizes. There is also a product line of jumbo HSS in sizes up to 32-in. square and thicknesses up to 2.36 in. For thicknesses up to 1 in., the sections are manufactured with ERW seams. For greater thicknesses, submerged arc welding (SAW) is used for the seams. SAW box sections can be produced from plate material (such as ASTM A572 Grade 50 as used in Example 3.3), but they generally are of a size that exceeds the 64-in. periphery limitation in ASTM A500. As such, these cross sections do not necessarily meet the requirements in ASTM A500. The specifier should contact the producer(s) of such cross sections to determine the cross sections that are made, as well as their cross-sectional properties and applicable production requirements. This information is also available in the EN10210 standard (CEN, 2006a, 2006b).

1.2 HSS Connection Design Standards and Scope


The scope of the AISC Specification, and hence this Design Guide, for HSS connections is limited to:
- Static design
- Single planar design
- Symmetry perpendicular to the plane (no offset elements)
- Unfilled and unreinforced HSS

Guidelines for conditions outside the scope of the AISC Specification appear in other codes and design guides:
- Fatigue (AWS, 2008; Zhao et al., 2001)
- Seismic design (Kurobane et al., 2004)
- Multiplanar connections (AWS, 2008; Packer and Henderson, 1997)
- Offset connections (AWS, 2008)
- Connections to concrete-filled HSS (Kurobane et al., 2004)

1.3 Advantages of HSS

HSS are very efficient sections for torsion and compression loading. For compressive loading, this is due to the favorable weak-axis radius of gyration, which often controls the available compressive strength.

It is not possible to make direct cost comparisons with other shapes because prices vary with time, application and geographic location. However, Table 1-2 compares two of the key factors that influence cost: weight and surface area that may require some type of preparation. The comparisons are between ASTM A992 wide-flange shapes, ASTM A500 Grade B round HSS, and ASTM A500 Grade B square HSS. Sections are selected for a particular length and load using AISC Manual Tables 4-1, 4-4 and 4-5. The load and resistance factor design (LRFD) load is 1.5 times the allowable strength design (ASD) load, and because this is the calibration load ratio in the AISC Specification, the same sections are determined in LRFD and ASD. The selected sections have comparable depths and available strengths. Data is provided for two cases: a compression member with moderate length (KL = 16 ft) and load, and a longer compression member (KL = 32 ft) with a higher load.

The wide-flange section is used as the basis for comparison and is assigned 100% for weight and surface area. It is apparent from Table 1-2 that these example HSS are 10 to 20% lighter than the corresponding wide-flange members, and have one-third to one-half less surface area. The latter is particularly influential for decreasing painting costs. Moreover, one should also bear in mind that this section comparison is performed using ASTM A500 Grade B material, which is readily available. If ASTM A500 Grade C material (with higher yield strength) were used, the advantage is even greater.
### Table 1-2. Comparison of W-Shape and HSS Compression Members

<table>
<thead>
<tr>
<th>Member type and grade</th>
<th>W-shape ASTM A992</th>
<th>HSS ASTM A500 Grade B</th>
<th>W-shape ASTM A992</th>
<th>HSS ASTM A500 Grade B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section</td>
<td>Round</td>
<td>Square</td>
<td>Round</td>
<td>Square</td>
</tr>
<tr>
<td>KL = 16 ft</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_a = 200$ kips (LRFD)</td>
<td>212 (LRFD)</td>
<td>208 (LRFD)</td>
<td>664 (LRFD)</td>
<td>620 (LRFD)</td>
</tr>
<tr>
<td>$P_a = 133$ kips (ASD)</td>
<td>141 (ASD)</td>
<td>138 (ASD)</td>
<td>442 (ASD)</td>
<td>412 (ASD)</td>
</tr>
<tr>
<td>KL = 32 ft</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_a = 600$ kips (LRFD)</td>
<td>600 (LRFD)</td>
<td>550 (LRFD)</td>
<td>734 (LRFD)</td>
<td>704 (LRFD)</td>
</tr>
<tr>
<td>$P_a = 400$ kips (ASD)</td>
<td>400 (ASD)</td>
<td>390 (ASD)</td>
<td>488 (ASD)</td>
<td>468 (ASD)</td>
</tr>
</tbody>
</table>

Note: The number in parentheses represents the percent of weight or surface area relative to the wide-flange option.

### 1.4 OTHER CONSIDERATIONS

#### 1.4.1 Notch Toughness

ASTM A500 (ASTM, 2007a) has no notch toughness requirements. However, ASTM A500 importantly notes “…Products manufactured to this specification may not be suitable for those applications such as dynamically loaded elements in welded structures, etc., where low-temperature notch-toughness properties may be important.” The new ASTM A501 Grade B includes a Charpy V-notch impact test with the minimum energy absorbed of 20 ft-lb (27 J) at a temperature of 0 °F (−18 °C) (ASTM, 2007b).

#### 1.4.2 Galvanizing Issues

In the ASTM A500 standard the outside corner radius of square and rectangular HSS is required to be equal to or less than 3t, where t is the wall thickness (ASTM, 2007a). The standard, ASTM A143/A143M-03 (ASTM, 2003) states that “a cold bending radius of three times the section thickness…will ordinarily ensure satisfactory properties in the final product.” Thus, all square and rectangular HSS produced in North America are potentially prone to corner cracking during hot-dip galvanizing because the outside corner radius averages 2t, and the upper limit on outside corner radius for all ASTM A500 HSS is at the minimum bending radius recommended for galvanizing. Kinstler’s (2005) report pointed out that “…the amount of cold work, as measured by the bending radius, is the most important single factor to consider when there is concern for brittle-type failure of steel galvanized after cold working.” ASTM A143/A143M-03 continues to advise that “…For heavy cold deformation exemplified by cold rolling…subcritical annealing at temperatures from 1200 to 1300 °F [650 to 705 °C] should be used.”

For hot-dip galvanizing, holes to allow for filling, venting and drainage are necessary, and adequate sizing minimizes differential thermal stresses experienced by the structure upon galvanizing. Vent holes with a minimum diameter of ½ in. (13 mm) and drain holes with a diameter of 1 in. (25 mm) are recommended.

#### 1.4.3 Internal Corrosion

The AISC Specification Commentary Section B3.11 discusses the topic of internal corrosion in HSS. In summary:

1. Corrosion will not occur in an enclosed building and is a consideration only in HSS exposed to weather.
2. Internal corrosion will not occur in a sealed HSS.
3. Pressure-equalizing holes at locations where water cannot flow into the HSS by gravity will prevent capillary action or aspiration through fine openings, such as an unwelded length of two plates or members in contact.
4. Internal protection is required only in open HSS where a change in air volume by ventilation or direct flow of water is possible, or when the HSS is subject to a temperature gradient that would cause condensation.

Care should be taken to keep water from remaining in the HSS during or after construction, because the expansion caused by freezing can create pressure sufficient to burst the HSS.
Chapter 2
Welding

AISC Design Guide 21, *Welded Connections—A Primer for Engineers* (Miller, 2006) provides excellent general advice for engineers on welding, including some particular remarks on welding HSS. With HSS-to-HSS welded connections, there are two design philosophies that can be used for weld design:

1. The weld can be proportioned so that it develops the yield strength of the connected branch wall at all locations around the branch. This will represent an upper limit on the weld size—and a conservative design procedure. This approach is particularly appropriate if plastic stress redistribution is required in the connection. The same effective weld size should be maintained all around the attached branch, except for the “hidden weld” in HSS-to-HSS partially overlapped K- or N-connections (see Chapter 8).

2. The weld can be proportioned so that it resists the applied forces in the branch. This approach is particularly appropriate if the branch forces are low relative to the branch member strength, such as when branches are sized for aesthetic reasons (e.g., the branches of a simply supported truss may be the same throughout the truss, yet, at the center of the truss, the web member forces will be very low under uniformly distributed load). If this design philosophy is adopted, effective weld lengths must be taken into account (see Section 2.4), and the same effective weld size should still be maintained all around the attached branch, with the entire branch perimeter welded (including the “hidden toe,” if applicable).

2.1 TYPES OF HSS WELDS

The four common types of welds used in HSS connections, in order of preference, are:

1. Fillet welds
2. Partial-joint-penetration (PJP) groove welds
3. Flare-bevel and flare-V-groove welds
4. Complete-joint-penetration (CJP) groove welds

2.1.1 Fillet Welds

Fillet welds are the most economical welds, and should be used in HSS connections whenever practical. However, for T-, Y-, K- and cross-connections (defined in Section 8.3) with fillet welds, it is recommended that the fillet weld strength enhancement, as given by AISC *Specification* Equation J2-5,
As the dihedral angle decreases from 90°, the theoretical throat increases. On the other side with the larger angle, the theoretical throat decreases. Table 2-1 provides the equivalent weld size factors to account for the skew angle. The actual weld size, \( w \), is multiplied by the factor to obtain the equivalent weld size, \( w_{eq} \), with the same throat and a 90° dihedral angle. On the side with the larger angle, the fillet weld size must be adjusted for the root opening, which cannot exceed \( \frac{3}{16} \) in. according to AWS D1.1/D1.1M (AWS, 2008) Section 5.22.1, if the attached element is cut square as shown in Figure 2-2. Example 2.1 shows the calculation of a skewed fillet weld.

### Table 2-1. Weld Size Factors for Skew

<table>
<thead>
<tr>
<th>Dihedral Angle (°)</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>1.41</td>
</tr>
<tr>
<td>70</td>
<td>1.23</td>
</tr>
<tr>
<td>80</td>
<td>1.10</td>
</tr>
<tr>
<td>90</td>
<td>1.00</td>
</tr>
<tr>
<td>100</td>
<td>0.923</td>
</tr>
<tr>
<td>110</td>
<td>0.863</td>
</tr>
<tr>
<td>120</td>
<td>0.816</td>
</tr>
<tr>
<td>130</td>
<td>0.780</td>
</tr>
</tbody>
</table>

2.1.2 PJP and CJP Groove Welds

In HSS-to-HSS connections, the welding can be done only from one side. Therefore, any groove-welded joint detail that requires preparation and welding on both sides—or backgouging—is not possible. PJP groove welds are preferable to CJP groove welds, when it is possible to use them. Prequalified PJP welded joints establish an effective throat, \( E \), as a function of the material thickness; weld preparation depth, \( S \); weld process; and position. The dimension \( E \) is equivalent to \( S \) minus any loss at the root due to lack of fusion. Design drawings should specify the required strength of the weld, and shop detail drawings must show the groove size, \( S \), and effective throat, \( E \).

When CJP groove welds are required, steel backing should be used, where possible, with a detailed root dimension to allow placement of sound weld metal in the full depth of the joint. In some cases, special inserts may be required to accommodate the HSS geometry. In statically loaded structures, this backing may be left in place.

In cyclically loaded structures, or other structures where backing cannot be left in place, an alternative approach is required, since it is not possible to remove the backing that is concealed inside the HSS. In such cases, CJP groove welds are also made with a backup weld at the root placed from the outside. Careful preparation of the joint is required to ensure the proper gap at the root. Because of the limited access and the skill required to fill the open root, often while out of position, a special 6GR certification is required for the welder (AWS, 2008).

Round HSS-to-HSS joints are complex and the bevel preparation varies around the perimeter of the joining member. A good discussion of various types of backing and joint details is given by Post (1990).

2.1.3 Flare-Bevel and Flare-V Groove Welds

Flare-bevel and flare-V groove welds are used at the rounded corners of rectangular and square HSS. They are used to attach flat elements that are wider than the HSS, or for matched HSS connections where the width of a branch member is the same as that of the continuous member in the connection. Flare-V groove welds are used when the corners of two square or rectangular HSS are welded together, such as in back-to-back, double-chord trusses. Figure 2-3 shows examples of flare-bevel and flare-V groove welds.

Table J2.2 of the AISC Specification gives the effective weld sizes for flare groove welds depending on the welding process, and presuming that the welds are filled flush with the outside face of the HSS. These are based on experimental research by Packer and Frater (2005), and take into account typical gap sizes that occur at the weld root. The effective weld sizes for flare-bevel groove welds are:

- \( \frac{4}{5}R \) for gas metal arc welding (GMAW)
- \( \frac{4}{5}R \) for flux-cored arc welding—gas shielded (FCAW-G)
- \( \frac{4}{5}R \) for flux-cored arc welding—self shielded (FCAW-S)
- \( \frac{4}{5}R \) for submerged arc welding (SAW)

\( R \) is the radius of the joint surface, which may be taken as 2\( t \) for HSS, giving an effective weld size of \( 1\frac{3}{4}t \) or \( \frac{5}{4}t \), respectively. If the weld is not filled flush to the HSS face (as in Figure 2-3), these effective throat sizes are reduced by subtracting the distance from the weld face to the HSS face.

Table J2.2 of the AISC Specification does not place an upper limit on \( R \) and is conservative in the resulting effective weld sizes for large \( R \) values. However, the AISC Specification does note that other effective throats may be used if demonstrated by tests. This information presumes that high heat input is applied (e.g., GMAW in full spray mode).

In matched box connections, a gap often exists between the inside of the branch and the corner of the continuous member, which makes it impossible to obtain a good root for the flare weld. The AISC Manual shows three methods for eliminating this problem (page 8-24): profile shaping across...
the width of the branch, placing a weld build-out at the corner along the length of the continuous member, and using a backing diaphragm inside the branch.

2.2 WELD INSPECTION

The AISC Manual lists and discusses (pages 8-4 to 8-7) the five most commonly used types of nondestructive testing (NDT) methods described in AWS D1.1 (AWS, 2008). The discussion is in general terms and applies to all types of welding. The following is a summary of NDT methods applied to HSS:

1. Visual testing (VT) must be performed on all welds (AWS, 2008) to ensure that they meet structural quality in HSS fabrication. All joints are examined prior to the commencement of welding to check fit-up, preparation of bevels, gaps, alignment and other variables. After the joint is welded, it is then visually inspected in accordance with AWS D1.1 (AWS, 2008). If a discontinuity is suspected, either the weld is repaired or other inspection methods are used to validate the integrity of the weld. In most cases, timely visual inspection by an experienced inspector is sufficient and offers the most practical and effective inspection alternative to other more costly methods.

2. Dye penetrant testing (PT) is useful for the detection of surface indications including tight cracks that are open to the surface. PT tends to be messy and slow, but it can be helpful when determining the extent of a defect found by visual examination. This is especially true when the defect is being removed by gouging or grinding for the repair of a weld to ensure that the defect is completely removed.

3. Magnetic particle testing (MT), as shown in Figure 2-4, is useful for the detection of surface indications and subsurface indication to a depth of approximately 0.1 in. MT can be used when the absence of cracking in areas of high restraint must be confirmed.

4. Ultrasonic testing (UT) has limited applicability in HSS fabrication. Relatively thin sections and variations in joint geometry can lead to difficulties in interpreting the signals, although technicians with specific experience on weldments similar to those to be examined may be able, in some instances, to decipher UT readings. Ultrasonic testing is not suitable for use with fillet welds and smaller PJP groove welds. CJP groove welds with and without backing bars also give readings that are subject to differing interpretations.

(a) Flare-bevel-groove weld between HSS corner and plate. The weld is not filled flush to the edge of the HSS.

(b) Flare-V-groove weld between two HSS corners. The weld is not filled flush to the edge of the HSS.

Fig. 2-3. Examples of flare-groove welds to square HSS (Packer and Frater, 2005). (Note the gap that typically occurs at the weld root.)

Fig. 2-4. Magnetic particle testing of an HSS connection.
5. Radiographic testing (RT) has very limited applicability in HSS fabrication because of the irregular shape of common joints and the resulting variations in thickness of material as projected onto film. RT can be used successfully for butt splices, but can only provide limited information about the condition of fusion at backing bars near the root corners. The general inability to place either the radiation source or the film inside the HSS means that exposures must usually be taken through both the front and back faces of the section with the film attached outside the back face. Several shots progressing around the member are needed to examine the complete joint.

2.3 EFFECTIVE SIZE OF FILLET WELDS

When an element is fillet-welded to an HSS, the AISC Specification requires that both the weld metal and the base metal be checked. With thin HSS, yielding or fracture of the base metal may be the critical limit state. When this is the case, the connection can be evaluated by using an effective weld size, which is the weld size that has the same strength as the base metal. Using this concept, the AISC Manual tables for eccentric loads on weld groups can be used to evaluate the base metal failure. It is also possible to establish minimum wall thicknesses required to develop the full strength of a particular weld size. Page 9-5 of the AISC Manual shows the minimum thickness to develop the rupture strength of a weld size \( D \) with \( F_{EXX} = 70 \) ksi. This concept can be expanded to include the yield strength, other strength filler metals, and an effective weld size for smaller thicknesses.

The nominal weld metal strength, per unit length, in AISC Specification Section J2.4 is:

\[
R_u = F_w \frac{D}{16\sqrt{2}} \tag{2-1}
\]

where

\[D = \text{number of sixteenths of an inch in the fillet weld size}\]

\[F_{EXX} = \text{electrode classification number, ksi}\]

The nominal shear strength of the base metal, per unit length next to the weld, in AISC Specification Section J4.2 is:

\[
R_u = F_{BM}t \tag{2-2}
\]

where

\[F_{BM} = 0.60F_y \text{ for shear yielding per AISC Specification Equation J4-3, or}\]

\[= 0.60F_u \text{ for shear rupture per AISC Specification Equation J4-4}\]

The effective weld size, \( D_{eff} \), is obtained by equating the available strength of the weld metal and the base metal. For shear yielding in the base metal:

\[
\begin{align*}
\phi_u F_u \left( \frac{D_{eff}}{16\sqrt{2}} \right) &= \phi_{BM} F_{BM}t \\
0.75 \left( 0.60 F_{EXX} \right) \left( \frac{D_{eff}}{16\sqrt{2}} \right) &= 0.75 \left( 0.60 F_y \right) t \\
D_{eff} &= 30.2 \left( \frac{F_u}{F_{EXX}} \right) t
\end{align*}
\]

For shear rupture in the base metal:

\[
\begin{align*}
\phi_u F_u \left( \frac{D_{eff}}{16\sqrt{2}} \right) &= \phi_{BM} F_{BM}t \\
0.60 F_{EXX} \left( \frac{D_{eff}}{16\sqrt{2}} \right) &= \frac{0.60 F_y t}{1.50} \\
D_{eff} &= 22.6 \left( \frac{F_u}{F_{EXX}} \right) t
\end{align*}
\]
### Table 2-2. Controlling Limit State for $D_{ef}$ for HSS Materials

<table>
<thead>
<tr>
<th>ASTM Material</th>
<th>$F_y$, ksi</th>
<th>$F_u$, ksi</th>
<th>$F_y/F_u$</th>
<th>Limit State</th>
<th>$D_{ef}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A53 Grade B</td>
<td>35</td>
<td>60</td>
<td>0.583</td>
<td>Base metal yielding</td>
<td>$30.2 \left( \frac{F_y}{F_{Exx}} \right) t$</td>
</tr>
<tr>
<td>Round</td>
<td>42</td>
<td>58</td>
<td>0.724</td>
<td>Base metal yielding</td>
<td>$30.2 \left( \frac{F_y}{F_{Exx}} \right) t$</td>
</tr>
<tr>
<td>Rectangular</td>
<td>46</td>
<td>58</td>
<td>0.793</td>
<td>Base metal rupture</td>
<td>$22.6 \left( \frac{F_u}{F_{Exx}} \right) t$</td>
</tr>
<tr>
<td>Grade C</td>
<td>46</td>
<td>62</td>
<td>0.742</td>
<td>Base metal yielding</td>
<td>$30.2 \left( \frac{F_y}{F_{Exx}} \right) t$</td>
</tr>
<tr>
<td>Round</td>
<td>50</td>
<td>62</td>
<td>0.806</td>
<td>Base metal rupture</td>
<td>$22.6 \left( \frac{F_u}{F_{Exx}} \right) t$</td>
</tr>
<tr>
<td>Rectangular</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 2-3. HSS Minimum Thickness, $t_{min}$, to Develop $F_{Exx}$ Strength, in.

<table>
<thead>
<tr>
<th>Weld Size, in.</th>
<th>A53 Grade B</th>
<th>A500 Grade B, Round</th>
<th>A500 Grade B, Rectangular</th>
<th>A500 Grade C, Round</th>
<th>A500 Grade C, Rectangular</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{16}$</td>
<td>0.199</td>
<td>0.166</td>
<td>0.160</td>
<td>0.151</td>
<td>0.150</td>
</tr>
<tr>
<td>$\frac{1}{8}$</td>
<td>0.265</td>
<td>0.221</td>
<td>0.214</td>
<td>0.202</td>
<td>0.200</td>
</tr>
<tr>
<td>$\frac{5}{32}$</td>
<td>0.331</td>
<td>0.276</td>
<td>0.267</td>
<td>0.252</td>
<td>0.250</td>
</tr>
<tr>
<td>$\frac{3}{32}$</td>
<td>0.397</td>
<td>0.331</td>
<td>0.320</td>
<td>0.302</td>
<td>0.300</td>
</tr>
<tr>
<td>$\frac{1}{16}$</td>
<td>0.464</td>
<td>0.386</td>
<td>0.374</td>
<td>0.353</td>
<td>0.350</td>
</tr>
<tr>
<td>$\frac{5}{64}$</td>
<td>0.530</td>
<td>0.442</td>
<td>0.427</td>
<td>0.403</td>
<td>0.400</td>
</tr>
<tr>
<td>$\frac{3}{64}$</td>
<td>0.596</td>
<td>0.497</td>
<td>0.481</td>
<td>0.453</td>
<td>0.450</td>
</tr>
</tbody>
</table>

The limit state for $D_{ef}$ for various HSS materials, and it can be seen that round products are controlled by base metal yielding, while square and rectangular products are controlled by base metal rupture.

By setting $D_{ef}$ equal to a particular weld size, the minimum HSS wall thickness, $t_{min}$, to develop the full strength of the weld can be determined. The matching filler metal for the various HSS grades has a strength level of 70 ksi. Table 2-3 gives the minimum HSS wall thickness to develop the 70-ksi-strength filler metal for fillet weld sizes typically used with HSS. This concept is based on one shear plane per weld. If there are two welds for one plate (e.g., a single-plate shear connection), each weld is associated with half the thickness of the plate.

In addition to the limit states associated with the weld, limit states associated with the HSS wall and the connecting element must be considered. Example 2.2 shows a calculation for the effective weld size.

---

### 2.4 EFFECTIVE WELD LENGTH

Due to the variation in flexural stiffness of the wall across the width of a rectangular HSS, a force transmitted through a weld is often not uniformly distributed. This can be accounted for in design using an effective length approach. This reduced effective length applies both to the weld and to the force in the connected element.

For elements transverse to an HSS face, the load is highest at the ends closest to the sidewalls of the HSS and decays to the smallest value at the center of the element. The net effect is that the weld may not be fully effective and could unzip with a crack starting at the ends. The AISC Specification accounts for this by specifying an effective weld length, $L_e$, to be used in calculating the available weld strength.

AISC Specification Equation K1-7 is used for welds to transverse plates, and it can be seen that the total effective weld length of Equation K1-7 is equal to twice (for two welds on each side of a plate) the effective width of a
transverse element given by \( b_{ov} \) in Equation K2-19. This illustrates the concept that where the philosophy of designing the welds in HSS connections to resist specific branch or plate loads is used, the effective weld lengths to be used can be taken as the effective widths of the connected elements considering the limit state of “local yielding due to uneven load distribution.” This effective length of transverse welds \( (L_e) \) and transverse elements \( (b_{ov}) \) dates back to research by Rolloos (1969); Wardenier et al. (1981); and Davies and Packer (1982). Attached elements (and welds) longitudinal (or parallel) to the axis of a square or rectangular HSS can be assumed to be fully effective; hence, the full weld length is used in Example 2.2.

In AISC Specification Section K2.3e, weld effective lengths, \( L_e \), are specified for welds to T-, Y-, cross- and gapped K-connections, which are defined in Section 8.3 of this Design Guide. These are simplified design recommendations that are generally more conservative than the foregoing \( b_{ov} \) method, which was based on transverse plates oriented at a branch angle of 90° to the HSS main (chord) member. The recommendations of Section K2.3e take into account the influence of the branch angle that was observed in experimental research on welds in HSS truss connections (Frater and Packer, 1992a, 1992b; Packer, 1995; Packer and Cassidy, 1995). Design Example 8.5 in this Design Guide demonstrates the effective weld length calculation for a K-connection.

No effective weld lengths are given in the AISC Specification for overlapped K-connections with square or rectangular members, but one can adopt the effective widths, \( b_{ov} \) and \( b_{ovv} \), in Section K2.3d for transverse welds in such connections. For square or rectangular HSS-to-HSS moment connections, one can similarly adopt the \( b_{ov} \) terms for effective widths of transverse welds, as given for transverse elements in Sections K3.3b(c) and K3.3c(c), as these represent the influence of local yielding due to uneven load distribution. Example 2.3 shows a calculation for a transverse weld to a square HSS.

### 2.5 WELDED JOINT DESIGN EXAMPLES

#### Example 2.1—Skewed Fillet Welds

**Given:**

Using the geometry illustrated in Figure 2-2, with \( \alpha = 20 \) and \( t_p = \frac{1}{2} \) in., determine the design strength (LRFD) and allowable strength (ASD) per unit length in shear for welds with size, \( w = \frac{1}{4} \) in., and 70-ksi filler metal. Also determine the adjustment to the weld size required for the left weld to account for the root opening that occurs due to the skew angle.

**Solution:**

For the right weld, the dihedral angle is 90° – 20° = 70°. From Table 2-1, the weld size factor is 1.23. Thus, the equivalent weld size, \( w_{eq} \), is:

\[
\begin{align*}
    w_{eq} &= \text{weld-size factor} \times w \\
    &= 1.23 \times \frac{1}{4} \text{ in.} \\
    &= 0.308 \text{ in.}
\end{align*}
\]

From AISC Specification Table J2.5, the nominal strength of the right weld per inch is:

\[
R_n = F_n A_w
\]

\[
= 42.0 \text{ ksi} \left( \frac{0.308 \text{ in.}}{\sqrt{2}} \right)
\]

\[
= 9.15 \text{ kips/in.}
\]

For the left weld, the dihedral angle is 90° + 20° = 110°. From Table 2-1, the weld size factor is 0.863. Thus, the equivalent weld size, \( w_{eq} \), is:

\[
\begin{align*}
    w_{eq} &= \text{weld-size factor} \times w \\
    &= 0.863 \times \frac{1}{4} \text{ in.} \\
    &= 0.216 \text{ in.}
\end{align*}
\]
From AISC Specification Table J2.5 and AISC Specification Equation J2-3, the nominal strength of the right weld per inch is:

\[ R_n = F_n A_w \]

\[ = 42.0 \text{ ksi} \left( \frac{0.216 \text{ in.}}{\sqrt{2}} \right) \]

\[ = 6.41 \text{ kips/in.} \]

The design strength (LRFD) and allowable strength (ASD) are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
</tbody>
</table>
| \( \phi R_n = 0.75 \left( 9.15 \text{ kips/in.} + 6.41 \text{ kips/in.} \right) \) | \( R_n = \frac{9.15 \text{ kips/in.} + 6.41 \text{ kips/in.}}{2.00} \)
| = 11.7 \text{ kips/in.} | = 7.78 \text{ kips/in.} |

The root opening that occurs due to the skew of the plate is:

\[ \text{root opening} = t_p \sin \alpha \]

\[ = \left( \frac{1}{2} \text{ in.} \right) \left( \sin 20^\circ \right) \]

\[ = 0.171 \text{ in.} < \frac{1}{16} \text{ in.} \quad \text{o.k.} \]

The adjusted weld size for the left weld is:

\[ \text{adjusted weld size} = w + \text{root opening} \]

\[ = \frac{1}{4} \text{ in.} + 0.171 \text{ in.} \]

\[ = 0.421 \text{ in.} \]

Use a \( \frac{3}{16} \)-in. fillet weld size (to account for the root opening).

**Example 2.2—Effective Weld Size**

**Given:**

Determine the 70-ksi weld size required to support the dead and live loads for the plate-to-Pipe connection shown in Figure 2-5. Also evaluate the capability of the Pipe wall to support the loads in direct shear.

From AISC Manual Tables 2-3 and 2-4, the material properties are as follows:

**Pipe 8 x-strong**

ASTM A53 Grade B

\( F_y = 35 \text{ ksi} \)

\( F_u = 60 \text{ ksi} \)

**PL \( \frac{3}{16} \times 9 \)**

ASTM A36

\( F_y = 36 \text{ ksi} \)

\( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-14, the Pipe geometric properties are as follows:

**Pipe 8 x-strong**

\( D = 8.63 \text{ in.} \)

\( t = 0.465 \text{ in.} \)
Solution:

Required strength

From Chapter 2 in *Minimum Design Loads for Buildings and Other Structures* (ASCE, 2006), hereafter referred to as ASCE 7, the required strength based on the vertical loads is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_e = 1.2(23.0 \text{ kips}) + 1.6(70.0 \text{ kips}) )</td>
<td>( P_e = 23.0 \text{ kips} + 70.0 \text{ kips} )</td>
</tr>
<tr>
<td>= 140 kips</td>
<td>= 93.0 kips</td>
</tr>
</tbody>
</table>

Weld size for the eccentric load on the weld group

For the eccentrically loaded weld group, from AISC Manual Table 8-4, with \( e = 3.00 \text{ in.} \) and \( l = 9.00 \text{ in.} \):

\[
a = \frac{e}{l} = \frac{3.00 \text{ in.}}{9.00 \text{ in.}} = 0.333
\]

For \( \theta = 0^\circ \) and the special case shown for the load not in the plane of the weld group, by linear interpolation using the values in the table for \( k = 0 \):

\[
C = 3.09 - 0.333(3.09 - 2.66)
= 2.95
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( D_{\min} = \frac{P_e}{\phi CC_l} )</td>
<td>( D_{\min} = \frac{\Omega P_e}{CC_l} )</td>
</tr>
<tr>
<td>= 140 kips</td>
<td>= ( \frac{2.00(93.0 \text{ kips})}{2.95(1.00)(9.00 \text{ in.})} )</td>
</tr>
<tr>
<td>= 0.75(2.95)(1.00)(9.00 \text{ in.})</td>
<td>= 7.03 \text{ sixteenths-of-an-inch}</td>
</tr>
<tr>
<td>= 7.03 \text{ sixteenths-of-an-inch}</td>
<td>= 7.01 \text{ sixteenths-of-an-inch}</td>
</tr>
</tbody>
</table>

Use \( \frac{1}{2} \)-in. fillet welds.

For the shear strength of the Pipe wall, the minimum thickness from Table 2-3 to develop \( \frac{1}{2} \)-in. fillet welds is 0.530 in., which exceeds the actual wall thickness, \( t = 0.465 \text{ in.} \). Therefore, the base metal controls.

Effective weld size

From Table 2-2, the effective weld size, \( D_{eff} \), is:

\[
D_{eff} = 30.2 \left( \frac{F_y}{F_{xx}} \right) t
= 30.2 \left( \frac{35 \text{ ksi}}{70 \text{ ksi}} \right)(0.465 \text{ in.})
= 7.02 \text{ sixteenths-of-an-inch}
\]
From AISC Manual Table 8-4, the design strength (LRFD) and the allowable strength (ASD) are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_e = \phi CC_i D_{eff} l )</td>
<td>( \frac{R_e}{\Omega} = \frac{CC_i D_{eff} l}{\Omega} )</td>
</tr>
<tr>
<td>( = 0.75 \left( 2.95 \text{ kips/in.} \right) \left( 1.00 \right) \left( 7.02 \right) \left( 9.00 \text{ in.} \right) )</td>
<td>( = \left( 2.95 \text{ kips/in.} \right) \left( 1.00 \right) \left( 7.02 \right) \left( 9.00 \text{ in.} \right) )</td>
</tr>
<tr>
<td>( = 140 \text{ kips} \geq 140 \text{ kips} \quad \text{o.k.} )</td>
<td>( = 93.0 \text{ kips} \geq 93.0 \text{ kips} \quad \text{o.k.} )</td>
</tr>
</tbody>
</table>

Note: The purpose of this example is to illustrate the effective weld size concept. In a complete connection design, other considerations, such as the punching shear limit state and its associated limits of applicability stipulated in AISC Specification Section K1, should also be checked. These considerations are addressed later in this Design Guide.

**Example 2.3—Transverse Weld to a Square or Rectangular HSS**

**Given:**

Determine the 70-ksi weld size required to support the dead and live loads for the transverse plate-to-HSS connection shown in Figure 2-6.

From AISC Manual Tables 2-3 and 2-4, the material properties are as follows:

- **HSS** 8×8×\( \frac{1}{2} \)  
  ASTM A500 Grade B  
  \( F_y = 46 \text{ ksi} \)  
  \( F_u = 58 \text{ ksi} \)

- **PL** 2×5\( \frac{1}{2} \)  
  ASTM A36  
  \( F_{yp} = 36 \text{ ksi} \)  
  \( F_{up} = 58 \text{ ksi} \)

From AISC Manual Table 1-12, the HSS geometric properties are as follows:

- **HSS** 8×8×\( \frac{1}{2} \)  
  \( B = 8.00 \text{ in.} \)  
  \( t = 0.465 \text{ in.} \)

The plate geometric properties are as follows:

- **PL** 1\( \frac{1}{2} \)×5\( \frac{1}{2} \)  
  \( B_p = 5.50 \text{ in.} \)  
  \( t_p = 0.500 \text{ in.} \)

**Fig. 2-6. Transverse welded plate to square HSS connection.**
Solution:

**Required strength**

From Chapter 2 in ASCE 7, the required strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u$</td>
<td>$1.2 (10.7 \text{ kips}) + 1.6 (32.6 \text{ kips})$</td>
<td>$10.7 \text{ kips} + 32.6 \text{ kips}$</td>
</tr>
<tr>
<td></td>
<td>$= 65.0 \text{ kips}$</td>
<td>$= 43.3 \text{ kips}$</td>
</tr>
</tbody>
</table>

**Effective weld length**

With fillet welds on both sides of the plate, from AISC Specification Section K1, the effective length, $L_e$, is:

\[
L_e = 2 \left[ \frac{10}{(B/t)} \right] \left[ \frac{F_f}{F_{y,bb}} \right] B_p \leq 2B_p
\]

\[
= 2 \left[ \frac{10}{(8.00 \text{ in.}/0.465 \text{ in.})} \right] \left[ \frac{46 \text{ ksi}(0.465 \text{ in.})}{36 \text{ ksi}(0.500 \text{ in.})} \right] (5.50 \text{ in.}) \leq 2(5.50 \text{ in.})
\]

\[
= 7.60 \text{ in.} < 11.0 \text{ in.}; \text{ therefore use } L_e = 7.60 \text{ in.}
\]

**Required weld size**

From AISC Specification Section J2.2a, the weld area, $A_w$, is:

\[
A_w = \frac{L_w}{\sqrt{2}}
\]

\[
= \frac{7.60 \text{ in.}(w)}{\sqrt{2}}
\]

\[
= (5.37 \text{ in.})(w)
\]

The nominal weld strength, $R_n$, is:

\[
R_n = F_w A_w
\]

\[
= 0.60 F_{exx} A_w
\]

\[
= 0.60 (70 \text{ ksi})(5.37 \text{ in.})(w)
\]

\[
= 226w \text{ kips/in.}
\]

Comparing design or allowable strength, as appropriate, to required strength, the required weld size is determined as follows:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_u$</td>
<td>$0.75$ from AISC Specification Table J2.5</td>
<td>$\Omega_w = 2.00$ from AISC Specification Table J2.5</td>
</tr>
<tr>
<td>$\phi_u R_w \geq P_u$</td>
<td>$\frac{R_w}{\Omega_w} \geq P_u$</td>
<td></td>
</tr>
<tr>
<td>$0.75 (226w \text{ kips/in.}) \geq 65.0 \text{ kips}$</td>
<td>$\frac{226w \text{ kips/in.}}{2.00} \geq 43.3 \text{ kips}$</td>
<td></td>
</tr>
<tr>
<td>$w \geq 0.383 \text{ in.}$</td>
<td>$w \geq 0.383 \text{ in.}$</td>
<td></td>
</tr>
</tbody>
</table>

Use \(\frac{3}{16}\)-in. fillet welds. The welds are placed over the full width on both sides of the plate, giving a total weld length of 11 in.

Note that the size of these welds accounts for the uneven force distribution across the width. If the total weld length were used in the calculation, a weld size of \(\frac{3}{8}\) in. would result—but would be inadequate to account for the uneven force distribution.
Chapter 3
Mechanical Fasteners

Bolting directly to HSS is difficult, due to lack of access to the interior of the member. ASTM A325 or A490 bolts can be used in a connection to an HSS wall if they are near an open end, or if access holes are cut in one of the other walls or in the bolted wall away from the connection to permit access for bolt installation inside the HSS. These access holes can be covered and sealed in a manner that reinforces the section and develops the original strength of the member. The welding of attachments to the HSS and then bolting to the attachments is also common.

Alternatively, Part 7 of the AISC Manual (see “Special Considerations for Hollow Structural Sections”) describes several types of mechanical fasteners that can be used to directly connect to the HSS wall. These include:

- Through-bolts
- Blind bolts
- Threaded studs
- Flow-drilled bolts
- Nails
- Screws

Section J3.10(c) in the AISC Specification provides design criteria for through-bolts, and threaded studs are covered in Table J3.2. The materials used for these connectors must conform to one of the material specifications in AISC Specification Section A3.4.

Fasteners are loaded in shear, tension or a combination of both, and the conventional connection design principles covered in the AISC Specification and Manual are applicable. In addition, Sections 3.1 and 3.2 cover the considerations for HSS walls for shear and tension applications, respectively. Examples of each are shown in Figure 3-1.

3.1 FASTENERS IN SHEAR

When the fasteners that connect directly to an HSS are loaded in shear, the design is essentially the same as for bolts that pass through a W-shape flange or web. HSS limit states are bolt bearing and block shear rupture if the connection is near the end of the HSS. Note that pin bearing strength equations are used instead of bolt bearing strength equations when through-bolting. Example 3.1 illustrates the design of a shear connection to an HSS column that has through-bolts loaded in shear.

3.2 FASTENERS IN TENSION

When the fasteners that connect directly to an HSS are loaded in tension, the design is similar to that for bolts that pass through a W-shape flange or web. However, two additional HSS limit states must be considered: pull-out through the wall of the HSS and distortion of the HSS wall.

The nominal strength for pull-out of a single fastener through an HSS wall is given by Packer and Henderson (1997) as:

\[ r_n = F_u(0.6\pi d_w t) \]

where \( d_w \) is the diameter of the part in contact with the inner surface of the HSS. A resistance factor, \( \phi \), of 0.67 for LRFD or a safety factor, \( \Omega \), of 2.25 for ASD is recommended for this limit state.

Fig. 3-1. Examples of direct fastening to HSS.

(a) A splice in a double chord truss where the bolts are loaded in shear, and have been installed with the aid of access holes.

(b) A beam-to-column end-plate moment connection where the blind bolts are loaded in tension.
3.3 BOLTED JOINT DESIGN EXAMPLES

Example 3.1—Through-Bolts in Shear

Given:
Determine the required bolt length and verify the adequacy of the through-bolt connection shown in Figure 3-2, with the dead and live loads, $P_D = 2.00$ kips and $P_L = 4.00$ kips, respectively.

From AISC Manual Table 2-3, the material properties are as follows:
- **HSS8×3×3/4**
  - ASTM A500 Grade B
  - $F_y = 46$ ksi
  - $F_u = 58$ ksi
- **L3½×3×3/4**
  - ASTM A36
  - $F_y = 36$ ksi
  - $F_u = 58$ ksi

From AISC Manual Table 1-11, the HSS geometric properties are as follows:
- **HSS8×3×3/4**
  - $H = 8.00$ in.
  - $B = 3.00$ in.
  - $t = 0.233$ in.

For distortion of the HSS wall, the criteria in Section K2 of the AISC Specification for chord wall plastification can be used. These criteria are based on a distortion limit when a yield-line mechanism first forms in the HSS wall, and can be applied to a fastener pattern transmitting a tension load to the HSS, by considering the fastener group as the branch of a T-connection with load assumed to act over an area that circumscribes the fasteners.

Example 3.2 illustrates the design of a connection to an HSS column that has threaded studs loaded in tension. Example 3.3 is similar, with bolts loaded in tension.

![Fig. 3-2. Through-bolt connection in shear.](image-url)
Solution:

Required bolt length

The required bolt length is calculated as the grip plus the thickness of any washers used plus the allowance specified in AISC Manual Table 7-15. The grip is the sum of the HSS depth and the angle thickness. The grip is:

\[ \text{Grip} = 3.00 \text{ in.} + \frac{1}{4} \text{ in.} = 3.25 \text{ in.} \]

Assuming one washer is used under the nut, the required bolt length is:

\[ L = \text{Grip} + \text{washer thickness} + \text{allowance} \]
\[ = 3.25 \text{ in.} + 0.177 \text{ in.} + \frac{1}{8} \text{ in.} = 4.30 \text{ in.} \]

Use \( L = 4.50 \text{ in.} \).

Required strength

From Chapter 2 in ASCE 7, the required strength of the connection is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_r = P_a )</td>
<td>( P_r = P_a )</td>
</tr>
<tr>
<td>( = 1.2 (2.00 \text{ kips}) + 1.6 (4.00 \text{ kips}) )</td>
<td>( = 2.00 \text{ kips} + 4.00 \text{ kips} )</td>
</tr>
<tr>
<td>( = 8.80 \text{ kips} )</td>
<td>( = 6.00 \text{ kips} )</td>
</tr>
</tbody>
</table>

Resultant load per bolt

The applied load is eccentric to the centroid of the bolt group with a 4-in. bolt spacing. AISC Manual Table 7-7 could be used with interpolation between the values obtained for a 3-in. and 6-in. spacing. Alternatively, the elastic method described in Part 7 of the AISC Manual can be used. The latter approach will be used here.

The eccentric load, \( P_r \), is resolved into a direct shear, \( V_v \), acting through the center of gravity of the bolt group and a moment, \( P_r e \), where \( e = 2.00 \text{ in.} \) is the eccentricity. The load per bolt due to direct shear is:

\[ V_v = \frac{P_r}{2} \]

The load per bolt due to the moment is:

\[ V_b = \frac{P_r e}{s} \]
\[ = \frac{P_r (2.00 \text{ in.})}{4.00 \text{ in.}} \]
\[ = 0.5 P_r \]

The resultant load per bolt is,

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_r = \sqrt{V_v^2 + V_b^2} )</td>
<td>( V_r = \sqrt{V_v^2 + V_b^2} )</td>
</tr>
<tr>
<td>( = 0.707 P_a )</td>
<td>( = 0.707 P_a )</td>
</tr>
<tr>
<td>( = 6.22 \text{ kips} )</td>
<td>( = 4.24 \text{ kips} )</td>
</tr>
</tbody>
</table>
Bolt shear strength

From AISC Manual Table 7-1, the bolt shear strength for a ½-in.-diameter ASTM A325 bolt in single shear with threads included in the shear plane is compared to the resultant load per bolt as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi, R = 11.0 \text{ kips} &gt; 6.22 \text{ kips} \quad \text{O.K.}$</td>
<td>$R / \Omega = 7.36 \text{ kips} &gt; 4.24 \text{ kips} \quad \text{O.K.}$</td>
</tr>
</tbody>
</table>

Bearing strength

For the bearing strength of the HSS wall and the angle, the grip is not restrained against expansion because of the through-bolt configuration. Therefore, as required in AISC Specification Section J3.10(c), the bearing strength should be checked using the provisions for pins in Section J7 of the AISC Specification.

For the HSS wall,

$R = 1.8F_y A_{pb}$

$= 1.8F_y t_d$

$= 1.8(46 \text{ ksi})(0.233 \text{ in.})(\frac{1}{8} \text{ in.})$

$= 12.1 \text{ kips}$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R = 0.75 \left(12.1 \text{ kips}\right)$</td>
<td>$R / \Omega = 12.1 \text{ kips}$</td>
</tr>
<tr>
<td>$= 9.08 \text{ kips} &gt; 6.22 \text{ kips} \quad \text{O.K.}$</td>
<td>$= 6.05 \text{ kips} &gt; 4.24 \text{ kips} \quad \text{O.K.}$</td>
</tr>
</tbody>
</table>

For the angle,

$R = 1.8F_y A_{pb}$

$= 1.8F_y t_d$

$= 1.8(36 \text{ ksi})(\frac{1}{4} \text{ in.})(\frac{1}{8} \text{ in.})$

$= 10.1 \text{ kips}$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R = 0.75 \left(10.1 \text{ kips}\right)$</td>
<td>$R / \Omega = 10.1 \text{ kips}$</td>
</tr>
<tr>
<td>$= 7.58 \text{ kips} &gt; 6.22 \text{ kips} \quad \text{O.K.}$</td>
<td>$= 5.05 \text{ kips} &gt; 4.24 \text{ kips} \quad \text{O.K.}$</td>
</tr>
</tbody>
</table>

Note: a complete design of this connection will also include checks of the angle gross and net section and block shear rupture strength. These checks are omitted for brevity in this example.

Example 3.2—Threaded Studs in Tension

Given:

Verify the adequacy of the connection shown in Figure 3-3. The connection consists of three ½-in.-diameter threaded studs welded to the HSS spaced 2 in. apart. Assume that the channel web thickness is sufficient to prevent any reduction in the stud strength due to prying action. The total dead and live loads are $P_d = 3.00 \text{ kips}$ and $P_L = 9.00 \text{ kips}$, respectively. Assume that there is no axial load in the HSS.
From AISC Manual Table 2-3, the material properties are as follows:

HSS8×8×\(\frac{3}{8}\)
ASTM A500 Grade B
\(F_y = 46\) ksi
\(F_u = 58\) ksi

For the \(\frac{1}{2}\)-in. studs, the material is ASTM A307 with \(F_u = 60\) ksi.

From AISC Manual Tables 1-12, the HSS geometric properties are as follows:

HSS8×8×\(\frac{3}{8}\)
\(B = 8.00\) in.
\(H = 8.00\) in.
\(t = 0.581\) in.

Solution:

Required strength

From Chapter 2 in ASCE 7, the required strength of the connection is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_u = 1.2\left(3.00\text{ kips}\right) + 1.6\left(9.00\text{ kips}\right))</td>
<td>(P_u = 3.00\text{ kips} + 9.00\text{ kips})</td>
</tr>
<tr>
<td>= 18.0 kips</td>
<td>= 12.0 kips</td>
</tr>
</tbody>
</table>

Tension strength of the three studs

From AISC Specification Section J3.6 and Table J3.2 (threaded parts), the nominal tension strength of each stud is:

\[
r_n = F_u A_n = 0.75 F_u \frac{\pi d^2}{4} = 0.75(60\text{ ksi}) \frac{\pi \left(\frac{1}{2}\right)^2}{4} = 8.84\text{ kips}
\]  

Fig. 3-3. Threaded stud-to-HSS connection in tension.
For three studs, the available tensile strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi \Phi = 3(\Phi_r)$</td>
<td>$R = 3\left(\frac{r}{\Omega}\right)$</td>
</tr>
<tr>
<td>$= 3(0.75)(8.84 \text{ kips})$</td>
<td>=$\frac{3(8.84 \text{ kips})}{2.00}$</td>
</tr>
<tr>
<td>$= 19.9 \text{ kips &gt; 18.0 kips o.k.}$</td>
<td>=$13.3 \text{ kips &gt; 12.0 kips o.k.}$</td>
</tr>
</tbody>
</table>

**Stud pull-out strength**

For pull-out of a single stud due to tension, the approach given in Section 3.2 can be adapted. There is no fastener head in contact with the inner surface, and pull-out is due to shear through the HSS wall resulting from the tension through the weld around the stud perimeter; $d_e$ can be taken as the diameter of the stud in this case. The pull-out strength of each stud is,

$$r_e = F_s \frac{0.60d_e}{t}$$

$$=(58 \text{ ksi})(0.6\pi)(\frac{1}{2} \text{ in.})(0.581 \text{ in.})$$

$$=31.8 \text{ kips}$$

For three studs, the available tensile strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.67$</td>
<td>$\Omega = 2.25$</td>
</tr>
<tr>
<td>$\phi \Phi = 3(\Phi_r)$</td>
<td>$R = 3\left(\frac{r}{\Omega}\right)$</td>
</tr>
<tr>
<td>$= 3(0.67)(31.8 \text{ kips})$</td>
<td>=$\frac{3(31.8 \text{ kips})}{2.25}$</td>
</tr>
<tr>
<td>$= 63.9 \text{ kips &gt; 18.0 kips o.k.}$</td>
<td>=$42.4 \text{ kips &gt; 12.0 kips o.k.}$</td>
</tr>
</tbody>
</table>

**HSS wall distortion strength (plastification)**

For the HSS wall distortion strength, the approach given in Section 3.2 can be used. The yield-line criterion for T-connections in the AISC *Specification* Section K2.3b and Equation K2-13 is used with the yield-line pattern shown in Figure C-K2.3(a) of the AISC *Specification Commentary* and Figure 8-2(a) in this Design Guide.

The effective dimensions of the “solid branch,” as shown in Figure 3-4 are:

$$B_e = 2(2.00 \text{ in.}) + 2\left(\frac{1}{2} \text{ in.}\right)$$

$$= 4.50 \text{ in.}$$

$$H_e = \text{stud diameter}$$

$$= \frac{1}{2} \text{ in.}$$

Equation K2-13 of the AISC *Specification* is a general yield-line solution and is deemed applicable for situations where $\beta \leq 0.85$.

In this case,

$$\beta = \frac{B_e}{B}$$

$$= 0.563 \leq 0.85 \text{ o.k.}$$
Additional limits on the connection configuration provided in Section K2.3a of the AISC Specification are:

\[
\frac{B}{t} = \frac{8.00}{0.581} = 13.8 \leq 35 \quad \text{o.k.}
\]

\[
F_y = 46 \text{ ksi} < 52 \text{ ksi} \quad \text{o.k.}
\]

\[
\frac{F_y}{F_u} = \frac{46}{58} = 0.793 < 0.8 \quad \text{o.k.}
\]

From AISC Specification Equation K2-13, with \( Q_f = 1.0 \) because there is no axial load in the HSS:

\[
\eta = \frac{H_b}{B} = \frac{\frac{1}{2}}{8.00} = 0.0625
\]

\[
P_n \sin \theta = F_y t \left[ \frac{2\eta}{(1-\beta)} + \frac{4}{\sqrt{1-\beta}} \right] Q_f
\]

\[
P_n = 46 \text{ ksi} \left( 0.581 \text{ in.} \right)^2 \left[ \frac{2(0.0625)}{(1-0.563)} + \frac{4}{\sqrt{1-0.563}} \right] 1.0 \sin 90^\circ
\]

\[
P_n = 98.4 \text{ kips}
\]

The available strength of the connection for the limit state of HSS wall distortion (plastification) is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_n = 1.00(98.4 \text{ kips}) )</td>
<td>( R_n = 98.4 \text{ kips} )</td>
</tr>
<tr>
<td>= 98.4 kips &gt; 18.0 kips ( \text{o.k.} )</td>
<td>( \Omega = 1.50 )</td>
</tr>
</tbody>
</table>

Note: A complete design of this connection will also include a check of the channel web in flexure. This check is omitted.

![Fig. 3-4. Yield line pattern of tension branch.](image)
EXAMPLE 3.3—BOLTS IN TENSION

Given:

Determine the available tensile strength, \( T_c \), of the WT5×22.5 made from ASTM A992 material bolted to an SAW18×18×3/8 box section made from ASTM A572 Grade 50 material as shown in Figure 3-5. The bolts are 3/8-in.-diameter ASTM A325 bolts in standard holes. Note that the design thickness for SAW box sections is the actual wall thickness of the member.

(a) Solve the problem assuming prying action does not limit the strength of the joint.

(b) Determine how prying action impacts the strength of the joint determined in Solution (a).

From AISC *Manual* Table 2-3, the material properties are as follows:

SAW18×18×3/8 box
ASTM A572 Grade 50
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

WT5×22.5
ASTM A992
\( F_y = 50 \text{ ksi} \)
\( F_u = 65 \text{ ksi} \)

From manufacturer’s literature and AISC *Manual* Table 1-8, the box section and WT geometric properties are as follows:

SAW18×18×3/8 box

\( B = 18.0 \text{ in.} \)
\( H = 18.0 \text{ in.} \)
\( t = 0.625 \text{ in.} \)
\( A = 41.4 \text{ in.}^2 \)

WT5×22.5

\( t_e = 0.350 \text{ in.} \)
\( b_f = 8.02 \text{ in.} \)
\( t_f = 0.620 \text{ in.} \)

![Diagram of bolted WT to SAW box section connection in tension.]

*Fig. 3-5. Bolted WT to SAW box section connection in tension.*
The connection has $\frac{3}{8}$-in.-diameter ASTM A563 nuts inside the box section. From AISC Manual Table 7-15, the width across flats, $W = 1\frac{1}{16}$ in. The gage, $g = 5.50$ in.

**Solution (a):**

**Bolt tensile strength**

From AISC Manual Table 7-2 (which is based upon AISC Specification Section J3.6), the available tensile strength per bolt for the limit state of tensile rupture, without consideration of prying action, is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi r_n = 20.7$ kips</td>
<td></td>
<td>$r_n = 13.8$ kips</td>
</tr>
<tr>
<td>For the group of four bolts:</td>
<td></td>
<td>For the group of four bolts:</td>
</tr>
<tr>
<td>$T_c = 4(\phi r_n)$</td>
<td>$= 4(20.7$ kips)</td>
<td>$= 4\left(\frac{r_n}{\Omega}\right)$</td>
</tr>
<tr>
<td></td>
<td>$= 82.8$ kips</td>
<td>$= 4(13.8$ kips)</td>
</tr>
</tbody>
</table>

**Bolt pull-out strength**

For pull-out of a single bolt due to tension, the approach given in Section 3.2 can be adapted. In this case, $d_n$ can be taken as the width of the nut, $W$. The pull-out strength of each bolt is:

$$r_n = F_n \left(0.6\pi d_n t\right)$$

$$= (65 \text{ ksi}) (0.6\pi) (1\frac{1}{16} \text{ in.}) (0.625 \text{ in.})$$

$$= 81.4 \text{ kips}$$

For four bolts, the available strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.67$</td>
<td></td>
<td>$\Omega = 2.25$</td>
</tr>
<tr>
<td>$T_c = 4(\phi r_n)$</td>
<td>$= 4(0.67)(81.4 \text{ kips})$</td>
<td>$T_c = 4\left(\frac{r_n}{\Omega}\right)$</td>
</tr>
<tr>
<td></td>
<td>$= 218$ kips</td>
<td>$= \frac{81.4 \text{ kips}}{2.25}$</td>
</tr>
</tbody>
</table>

**HSS wall distortion strength (plastification)**

For the HSS wall distortion strength, the approach given in Section 3.2 can be used by considering the loaded block of area on the HSS connecting face bounded by the four point loads (bolts in tension). The yield line criterion for T-connections is given in AISC Specification Section K2.3b. Equation K2-13 of the AISC Specification is a general yield-line solution that is deemed applicable for situations where $\beta \leq 0.85$. Conservatively, use $B_b = 5.50$ in. and $H_b = 6.00$ in. In this case,

$$\beta = \frac{B_b}{B}$$

$$= \frac{5.50}{18.0} \text{ in.}$$

$$= 0.306 \leq 0.85 \quad \text{o.k.}$$
Additional limits on the connection configuration provided in Section K2.3a of the AISC Specification are:

\[
B = \frac{18.0 \text{ in.}}{t} = \frac{18.0}{0.625} = 28.8 \leq 35 \quad \text{o.k.}
\]

\[
F_v = 50 \text{ ksi} < 52 \text{ ksi} \quad \text{o.k.}
\]

\[
\frac{F_v}{F_u} = \frac{50}{65} = 0.769 < 0.8 \quad \text{o.k.}
\]

\[
Q_f = 1.3 - 0.4 \frac{U}{\beta} \leq 1.0 \quad \text{for chord (connecting surface) in compression} \quad \text{(Spec. Eq. K2-10)}
\]

where

\[
U = \left( \frac{P_r}{AF_c} + \frac{M_r}{SF_c} \right) \quad \text{(Spec. Eq. K2-12)}
\]

From Chapter 2 in ASCE 7, the required axial strength, \( P_r \), is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_r = P_a ) = 1.2(200 kips) + 1.6(200 kips)</td>
<td>( P_r = P_a = 200 \text{ kips} + 200 \text{ kips} )</td>
</tr>
<tr>
<td>= 560 kips</td>
<td>= 400 kips</td>
</tr>
</tbody>
</table>

Because there is no moment in the column, \( M_r = 0 \).

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_c = F_y ) = 50 ksi</td>
<td>( F_c = 0.6F_y = 30.0 \text{ ksi} )</td>
</tr>
<tr>
<td>( U = \frac{560 \text{ kips}}{41.4 \text{ in.}^2 (50 \text{ ksi})} + 0 ) = 0.271</td>
<td>( U = \frac{400 \text{ kips}}{41.4 \text{ in.}^2 (30.0 \text{ ksi})} + 0 ) = 0.322</td>
</tr>
<tr>
<td>( Q_f = 1.3 - 0.4 \left( \frac{0.271}{0.306} \right) ) = 0.946</td>
<td>( Q_f = 1.3 - 0.4 \left( \frac{0.322}{0.306} \right) ) = 0.879</td>
</tr>
</tbody>
</table>

The nominal strength is determined from,

\[
P_r \sin \theta = F_y t^2 \left[ \frac{2\eta}{(1 - \beta)} + \frac{4}{\sqrt{1 - \beta}} \right] Q_f \quad \text{(Spec. Eq. K2-13)}
\]

where

\[
\eta = \frac{H_b}{B} = \frac{6.00 \text{ in.}}{18.0 \text{ in.}} = 0.333
\]
The available strength for the limit state of HSS wall distortion (plastification) is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 1.00$</td>
<td>$\Omega = 1.50$</td>
</tr>
<tr>
<td>$P_n = 50 \text{ ksi (0.625 in.)}^2 \times \left[ \frac{2(0.333)}{1-0.306} + \frac{4}{\sqrt{1-0.306}} \right] \frac{0.946}{\sin 90^\circ}$</td>
<td>$P_n = 50 \text{ ksi (0.625 in.)}^2 \times \left[ \frac{2(0.333)}{1-0.306} + \frac{4}{\sqrt{1-0.306}} \right] \frac{0.879}{\sin 90^\circ}$</td>
</tr>
<tr>
<td>= 106 kips</td>
<td>= 98.9 kips</td>
</tr>
<tr>
<td>$T_c = \phi P_n$</td>
<td>$T_c = \frac{P_n}{\Omega}$</td>
</tr>
<tr>
<td>= 1.00(106 kips)</td>
<td>= $\frac{98.9}{1.50}$</td>
</tr>
<tr>
<td>= 106 kips</td>
<td>= 65.9 kips</td>
</tr>
</tbody>
</table>

Thus, the bolt strength governs with the available tensile strength, $T_c = 82.8$ kips for LRFD, and $T_c = 55.2$ kips for ASD.

**Solution (b):**

In Solution (a) it was determined that bolt tensile strength governed. For the calculated available strengths to be valid, the tee flange thickness must satisfy the minimum thickness required to eliminate prying action (see page 9-10 in the AISC Manual) at a load equal to the calculated bolt tension strength.

$$b = \text{distance from bolt centerline to face of tee web, in.}$$
$$= \frac{g - t_s}{2}$$
$$= \frac{5.50 \text{ in.} - 0.350 \text{ in.}}{2}$$
$$= 2.58 \text{ in.}$$

$$b' = \text{distance from face of bolt to face of tee web, in.}$$
$$= b - \frac{d_s}{2}$$
$$= 2.58 \text{ in.} - \frac{5/8 \text{ in.}}{2}$$
$$= 2.27 \text{ in.}$$

The tributary width, $p$, is:

$$p = \frac{8 \text{ in.}}{2}$$
$$= 4.00 \text{ in.}$$

According to page 9-11 of the AISC Manual, $p$ should preferably not exceed the gage, $g$, equal to 5.50 in. in this case. This limit is larger and therefore does not control in this case.

Using the bolt tensile strength, the thickness required to eliminate prying action (with the bolt available strength equal to $\phi r_n = 20.7$ kips for LRFD and $r_n/\Omega = 13.8$ kips for ASD) is:
Because \( t_{\text{min}} > t_f \), prying action does reduce the strength of the bolts and therefore must be considered. Using the procedure provided on page 9-12 of the AISC Manual for the calculation of \( Q \):

\[
t = t_f \]
\[
= 0.620 \text{ in.}
\]
\[
t_c = t_{\text{min}}
\]
\[
= 0.896 \text{ in.}
\]
\[
d' = d_h
\]
\[
= \frac{11}{16} \text{ in.}
\]
\[
\delta = 1 - \frac{d'}{p}
\]
\[
= 1 - \frac{\frac{11}{16} \text{ in.}}{4.00 \text{ in.}}
\]
\[
= 0.828
\]
\[
a = \frac{b_f - g}{2}
\]
\[
= \frac{8.02 \text{ in.} - 5.50 \text{ in.}}{2}
\]
\[
= 1.26 \text{ in.}
\]
\[
a' = a + \frac{d_h}{2} \leq 1.25b + \frac{d_h}{2}
\]
\[
= 1.26 \text{ in.} + \frac{\frac{11}{16} \text{ in.}}{2} \leq 1.25 \left( 2.58 \text{ in.} \right) + \frac{\frac{11}{16} \text{ in.}}{2}
\]
\[
= 1.57 \text{ in.} \leq 3.54 \text{ in.}
\]

Therefore, \( a' = 1.57 \text{ in.} \)

\[
\rho = \frac{b'}{a'}
\]
\[
= \frac{2.27 \text{ in.}}{1.57 \text{ in.}}
\]
\[
= 1.45
\]
\[ \alpha' = \frac{1}{\delta(1+\rho)} \left[ \left( \frac{t_c}{t} \right)^2 - 1 \right] \]
\[ = \left( \frac{1}{0.828(1+1.45)} \right) \left[ \left( \frac{0.896 \text{ in.}}{0.620 \text{ in.}} \right)^2 - 1 \right] \]
\[ = 0.537 \]

With \( 0 \leq \alpha' \leq 1 \):
\[ Q = \left( \frac{t}{t_c} \right)^2 (1 + \delta \alpha') \]
\[ = \left( \frac{0.620 \text{ in.}}{0.896 \text{ in.}} \right)^2 \left[ 1 + 0.828 \left( 0.537 \right) \right] \]
\[ = 0.692 \]

Thus, for all four bolts:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = 4 (\phi r_c)Q )</td>
<td>( \frac{R_n}{\Omega} = 4 \left( \frac{r_c}{\Omega} \right)Q )</td>
</tr>
<tr>
<td>( = 4 \times (20.7 \text{ kips}) \times (0.692) )</td>
<td>( = 4 \times (13.8 \text{ kips}) \times (0.692) )</td>
</tr>
<tr>
<td>( = 57.3 \text{ kips} )</td>
<td>( = 38.2 \text{ kips} )</td>
</tr>
</tbody>
</table>

Comparing to Solution (a), prying action reduces the available strength by approximately 30\%.
Chapter 4
Moment Connections

4.1 W-BEAMS TO HSS COLUMNS

Contemporary information concerning moment connections to HSS columns is limited in literature, although some recommendations appear in design guides such as by Packer and Henderson (1997) and Kurobane et al. (2004). Part 12 of the AISC Manual discusses seven types of these connections:

- HSS through-plate flange-plated
- HSS cut-out plate flange-plated
- HSS directly welded
- HSS end plate
- HSS above and below continuous beams
- HSS welded tee flange
- HSS diaphragm plate

As with this entire Design Guide, the scope is restricted to nonseismic applications. Note that HSS-to-HSS moment connections are covered in Chapter 9 of this Design Guide.

The best configuration for a particular application depends on four factors:

1. The magnitude of the moment that must be transferred to the HSS
2. The magnitude of the moment that must be transferred through the HSS
3. The magnitude of the axial force in the HSS
4. The requirements for orthogonal framing

The detail of the connection must accommodate mill tolerances of the W-beam. These include variations in the depth and tilt of the flanges. Shimming in the field with conventional shims or finger shims is the most commonly used method. The examples in this chapter illustrate the limit states that must be considered in design.

4.2 CONTINUOUS BEAM OVER HSS COLUMN

A continuous beam over an HSS column, as shown in Figure 4-1, is a very efficient connection for single-story construction. However, there is an important stability issue that must be considered. In this type of construction, the beam is braced only at the top flange. Therefore, out-of-plane motion at the top of the HSS is restrained only by the relatively weak flexural stiffness of the beam web, greatly increasing the effective length of the column.

The restraint to this motion must be increased by adding stiffeners to the web, as in Figure 4-1(a), or by using a bottom chord extension to the joist supported by the beam at the connection as in Figure 4-1(b), or both, as illustrated in Figure 4-1(c). The stiffener in (a) can also be used as the orthogonal framing connection. See the AISC Manual Part 2 for additional information and details for beams framing continuously over columns.

AISC Manual CD Companion Example K.11 (AISC, 2005c) is a continuous beam over an HSS column, but only the limit states associated with a concentrated load on the end of an HSS are evaluated. Example 4.1 in Section 4.5 examines all the limit states.

![Fig. 4-1. Beam over HSS column connection.](image-url)
4.3 THROUGH-PLATE CONNECTIONS

If the required moment transfer to the column is larger than can be provided by a bolted base plate or cap plate, or if the HSS width is larger than that of the W-shape beam, a through-plate fully-restrained (FR) connection can be used. Example 4.2 shows a calculation for a through-plate connection.

4.4 DIRECTLY WELDED CONNECTIONS

Some moment transfer can be achieved by welding the flanges of the W-shape directly to the wall of the HSS column as shown in Figure 4-2. The single-plate connection transfers all of the shear force and facilitates erection of the beam. These connections may be capable of developing the full flexural strength of the HSS (depending primarily on the beam width relative to the HSS column width). The full flexural strength of the W-shape, however, is seldom achievable.

The strength of the flanges in tension and compression are calculated by considering them as transverse plates acting on the face of the HSS column. To obtain the maximum efficiency, the HSS should be thick, and the beam flange width should be close to the flat width of the HSS, estimated as \( B - 3t \).

The limit states that must be considered in AISC Specification Section K1.3b are local yielding of the beam flange due to uneven load distribution, HSS shear yielding (punching), sidewall tension, and sidewall compression failure. However, shear yielding only needs to be checked for certain flange widths and the sidewall limit states only apply when the flange and HSS widths are the same. This is also tabulated in Table 7-2 of this Design Guide. Example 4.3 shows a calculation for a directly welded connection.

![Fig. 4-2. Directly welded connection.](image-url)
4.5 CONNECTION DESIGN EXAMPLES

Example 4.1—Beam over HSS Column Connection

Given:
Verify the adequacy of the connection shown in Figure 4-3, between a two-span W18×40 beam supported by an HSS8×8×⅛ column. The column axial dead and live loads are $P_D = 7.50$ kips and $P_L = 22.5$ kips, respectively. The moment transferred across the beam-to-column interface consists of dead and live load moments of $M_D = 4.50$ kip-ft and $M_L = 13.5$ kip-ft, respectively.

From AISC Manual Tables 2-3 and 2-4, the material properties are as follows:

**HSS8×8×⅛**
ASTM A500 Grade B
$F_y = 46$ ksi
$F_u = 58$ ksi

**W18×40**
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

**Cap plate**
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From AISC Manual Table 1-12, the HSS geometric properties are as follows:

**HSS8×8×⅛**
$H = 8.00$ in.
$B = 8.00$ in.
$t = 0.233$ in.

---

Fig. 4-3. W-beam over HSS column connection.
From AISC Manual Table 1-1, the W-shape geometric properties are as follows:

- **W18×40**
  - \( d = 17.9 \) in.
  - \( t_w = 0.315 \) in.
  - \( t_f = 0.525 \) in.
  - \( k = 0.927 \) in.
  - \( k_1 = \frac{1}{16} \) in.

The gage, \( g \), is 3.50 in. for the W18×40.

The geometric properties of the cap plate and bolts are as follows:

- **Cap plate**
  - length = 14.0 in.
  - width = 9.00 in.
  - \( t_p = 0.500 \) in.

- **ASTM A325 bolts**
  - \( d_b = \frac{3}{8} \) in.

**Solution:**

From Chapter 2 of ASCE 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_a = 1.2 \left( 7.50 \text{ kips} \right) + 1.6 \left( 22.5 \text{ kips} \right) )</td>
<td>( P_a = 7.50 \text{ kips} + 22.5 \text{ kips} )</td>
</tr>
<tr>
<td>= 45.0 kips</td>
<td>= 30.0 kips</td>
</tr>
<tr>
<td>( M_a = 1.2 \left( 4.50 \text{ kip-ft} \right) + 1.6 \left( 13.5 \text{ kip-ft} \right) )</td>
<td>( M_a = 4.50 \text{ kip-ft} + 13.5 \text{ kip-ft} )</td>
</tr>
<tr>
<td>= 27.0 kip-ft</td>
<td>= 18.0 kip-ft</td>
</tr>
</tbody>
</table>

**Tensile load in bolt**

Calculate the bolt tension, \( T_a \), assuming that the compression force, \( C_r \), acts at the face of the column as shown in Figure 4-3(a).

From the summation of forces and moments with \( C_r = C_a \) and \( T_r = T_u \) for LRFD, and \( C_r = C_a \) and \( T_r = T_u \) for ASD, the required bolt tension is determined as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_r = C_a; T_r = T_u )</td>
<td>( C_r = C_a; T_r = T_u )</td>
</tr>
<tr>
<td>( P_a = 45.0 \text{ kips} )</td>
<td>( P_a = 30.0 \text{ kips} )</td>
</tr>
<tr>
<td>( = C_a - 2T_u )</td>
<td>( = C_a - 2T_u )</td>
</tr>
<tr>
<td>( M_a = 27.0 \text{ kip-ft} \left( 12 \text{ in./ft} \right) )</td>
<td>( M_a = 18.0 \text{ kip-ft} \left( 12 \text{ in./ft} \right) )</td>
</tr>
<tr>
<td>( = 324 \text{ kip-in.} )</td>
<td>( = 216 \text{ kip-in.} )</td>
</tr>
<tr>
<td>( M_a = \frac{8.00 \text{ in.}}{2} C_a + \frac{11.0 \text{ in.}}{2} \left( 2T_u \right) )</td>
<td>( M_a = \frac{8.00 \text{ in.}}{2} C_a + \frac{11.0 \text{ in.}}{2} \left( 2T_u \right) )</td>
</tr>
<tr>
<td>( 324 \text{ kip-in.} = 4.00 \text{ in.} \left( 45.0 \text{ kips} + 2T_u \right) + 5.50 \text{ in.} \left( 2T_u \right) )</td>
<td>( 216 \text{ kip-in.} = 4.00 \text{ in.} \left( 30.0 \text{ kips} + 2T_u \right) + 5.50 \text{ in.} \left( 2T_u \right) )</td>
</tr>
<tr>
<td>( 2T_u = 15.2 \text{ kips on 2 bolts} )</td>
<td>( 2T_u = 10.1 \text{ kips on 2 bolts} )</td>
</tr>
<tr>
<td>( T_u = \frac{15.2 \text{ kips}}{2} )</td>
<td>( T_u = \frac{10.1 \text{ kips}}{2} )</td>
</tr>
<tr>
<td>( = 7.60 \text{ kips/bolt} )</td>
<td>( = 5.05 \text{ kips/bolt} )</td>
</tr>
</tbody>
</table>
**Effect of prying action—W-shape flange**

Check the flange thickness for prying action per Part 9 of the AISC *Manual*.

Using Figure 4-3(b):

\[ b = \text{distance from bolt centerline to face of beam web} \]
\[ = \left( g - t_f \right) / 2 \]
\[ = \left( 3.50 \text{ in.} - 0.315 \text{ in.} \right) / 2 \]
\[ = 1.59 \text{ in.} \]

\[ b' = \text{distance from face of bolt to face of web} \]
\[ = b - \frac{d_b}{2} \]
\[ = 1.59 \text{ in.} - \frac{\frac{3}{8} \text{ in.}}{2} \]
\[ = 1.22 \text{ in.} \]

\[ p = \text{tributary length per pair of bolts in the perpendicular direction, which should preferably not exceed the gage between the pair of bolts, } g \]
\[ = g \]
\[ = 3.50 \text{ in.} \]

The following is a simplified check provided in Part 9 of the AISC *Manual* based upon the “no prying action” equation.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ t_{\text{min}} = \frac{4.44T_n b'}{p F_u} ]</td>
<td>[ t_{\text{min}} = \frac{6.66 T_n b'}{p F_u} ]</td>
</tr>
<tr>
<td>[ = \frac{4.44 \left( 7.60 \text{ kips/bolt} \right) \left( 1.22 \text{ in.} \right)}{3.50 \text{ in.} \left( 65 \text{ ksi} \right)} ]</td>
<td>[ = \frac{6.66 \left( 5.05 \text{ kips/bolt} \right) \left( 1.22 \text{ in.} \right)}{3.50 \text{ in.} \left( 65 \text{ ksi} \right)} ]</td>
</tr>
<tr>
<td>= 0.425 in.</td>
<td>= 0.425 in.</td>
</tr>
<tr>
<td>0.425 in. &lt; 0.525 in. o.k.</td>
<td>0.425 in. &lt; 0.525 in. o.k.</td>
</tr>
</tbody>
</table>

Because \( t_{\text{min}} < t_f \), there is no prying action in the beam flange.

**Effect of prying action—cap plate**

Check the cap plate thickness for prying action per Part 9 of the AISC *Manual*.

Using Figure 4-3(a):

\[ b = \text{distance from bolt centerline to face of HSS} \]
\[ = \left( 11.0 \text{ in.} - 8.00 \text{ in.} \right) / 2 \]
\[ = 1.50 \text{ in.} \]

\[ b' = \text{distance from face of bolt to face of HSS} \]
\[ = b - \frac{d_b}{2} \]
\[ = 1.50 \text{ in.} - \frac{\frac{3}{8} \text{ in.}}{2} \]
\[ = 1.13 \text{ in.} \]
\[ p = \text{tributary length per pair of bolts in the perpendicular direction, which should preferably not exceed the gage between the pair of bolts, } g \]

= half the cap plate width

= \frac{9.00 \text{ in.}}{2} 

= 4.50 \text{ in.} 

Use \( p = 4.50 \text{ in.} \)

The gage limitation given on page 9-11 of the AISC Manual does not apply to this configuration because with bending checked at the face of the HSS, and the “gage” being interrupted by the HSS, an artificially large value of \( g \) results.

The following is a simplified check provided in Part 9 of the AISC Manual based upon the “no prying action” equation.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ t_{\text{min}} = \frac{4.44T_a b'}{p F_{up}} = \frac{4.44(7.58 \text{ kips/bolt})(1.13 \text{ in.})}{4.50 \text{ in.}(58 \text{ ksi})} = 0.382 \text{ in.} ]</td>
<td>[ t_{\text{min}} = \frac{6.66T_a b'}{p F_{up}} = \frac{6.66(5.05 \text{ kips/bolt})(1.13 \text{ in.})}{4.50 \text{ in.}(58 \text{ ksi})} = 0.382 \text{ in.} ]</td>
</tr>
</tbody>
</table>

Because \( t_{\text{min}} < t_p \), there is no prying action in the cap plate.

**Bolt available tensile strength**

Because there is no prying action, the required strength of the bolt is \( T_a \) (\( T_a \) for LRFD and \( T_a \) for ASD). The available tensile strength, \( \phi r_a \) and \( r_a/\Omega \), of a 3/8-in.-diameter ASTM A325 bolt is given in AISC Manual Table 7-2.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi r_a \geq T_a = 7.60 \text{ kips/bolt} )</td>
<td>( r_a/\Omega \geq T_a = 5.05 \text{ kips/bolt} )</td>
</tr>
<tr>
<td>( \phi r_a = 29.8 \text{ kips/bolt} )</td>
<td>( r_a/\Omega = 19.9 \text{ kips/bolt} )</td>
</tr>
<tr>
<td>29.8 kips &gt; 7.60 kips \quad \text{o.k.}</td>
<td>19.9 kips &gt; 5.05 kips \quad \text{o.k.}</td>
</tr>
</tbody>
</table>

**Beam web local yielding**

The limit state of beam web local yielding is checked as follows.

Determine the compression force, \( C_r \), where \( C_r = C_u \) for LRFD and \( C_r = C_a \) for ASD.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ C_r = P_u + 2T_a = 45.0 \text{ kips} + 2(7.60 \text{ kips/bolt}) = 60.2 \text{ kips} ]</td>
<td>[ C_r = P_a + 2T_a = 30.0 \text{ kips} + 2(5.05 \text{ kips/bolt}) = 40.1 \text{ kips} ]</td>
</tr>
</tbody>
</table>

Assume a 2.5:1 dispersion slope or 21.8° dispersion angle of the load from the HSS wall through the cap plate. From the AISC Specification Commentary Section K1.6:

\[ N = t + 5t_p \]

\[ = 0.233 \text{ in.} + 5(0.500 \text{ in.}) \]

\[ = 2.73 \text{ in.} \]
From AISC Specification Section J10.2,
\[ R_k = [5k + N] F_{uw} \]
\[ = [5(0.927 \text{ in.}) + 2.73 \text{ in.}](50 \text{ ksi})(0.315 \text{ in.}) \]
\[ = 116 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 1.00 )</td>
<td></td>
</tr>
<tr>
<td>( \phi R_n = 1.00(116 \text{ kips}) = 116 \text{ kips} )</td>
<td></td>
</tr>
<tr>
<td>116 kips &gt; 60.2 kips o.k.</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td></td>
<td>( R_n/\Omega = \frac{116 \text{ kips}}{1.50} = 77.3 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>77.3 kips &gt; 40.1 kips o.k.</td>
</tr>
</tbody>
</table>

**Beam web crippling**

Check the limit state of beam web crippling using AISC Specification Section J10.3 and the following equation.

\[ R_n = 0.80 t_w^2 \left[ 1 + \frac{3N}{d} \left( \frac{t_w}{t_f} \right)^{1.5} \right] \sqrt{EF_{uw}t_f} \]

Using AISC Manual Table 9-4, this equation can be simplified as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>For ( N/d = 2.73 \text{ in.} /17.9 \text{ in.} = 0.153 \leq 0.2: )</td>
<td></td>
</tr>
<tr>
<td>( \phi R_n = 2\left[ \phi R_i + N\phi R_\theta \right] )</td>
<td></td>
</tr>
<tr>
<td>( \phi R_i = 46.3 \text{ kips} )</td>
<td></td>
</tr>
<tr>
<td>( \phi R_\theta = 3.60 \text{ kips/in.} )</td>
<td></td>
</tr>
<tr>
<td>( \phi R_n = 2\left[ 46.3 \text{ kips} + 2.73 \text{ in.}(3.60 \text{ kips/in.}) \right] )</td>
<td></td>
</tr>
<tr>
<td>112 kips &gt; 60.2 kips o.k.</td>
<td>For ( N/d = 2.73 \text{ in.} /17.9 \text{ in.} = 0.153 \leq 0.2: )</td>
</tr>
<tr>
<td>( R_n/\Omega = 2\left[ R_i/\Omega + N R_\theta/\Omega \right] )</td>
<td></td>
</tr>
<tr>
<td>( R_i/\Omega = 30.9 \text{ kips} )</td>
<td></td>
</tr>
<tr>
<td>( R_\theta/\Omega = 2.40 \text{ kips/in.} )</td>
<td></td>
</tr>
<tr>
<td>( R_n/\Omega = 2\left[ 30.9 \text{ kips} + 2.73 \text{ in.}(2.40 \text{ kips/in.}) \right] )</td>
<td></td>
</tr>
<tr>
<td>74.9 kips &gt; 40.1 kips o.k.</td>
<td></td>
</tr>
</tbody>
</table>

Note that if the total axial force were applied at the beam top flange over the connection, web compression buckling (AISC Specification Section J10.5) would also be checked to determine if a stiffener is required over the HSS wall.

**HSS wall strength**

The compression force in the HSS wall may not be uniformly distributed over the entire HSS wall width due to shear lag. This phenomenon is also discussed in Section 7.4 of this Design Guide. Assume that the force in the beam web disperses at a slope of 2.5:1 (or 21.8°). From Carden et al. (2008):

\[ N = 2k_\ell + 5t_f \]
\[ = 2(1/16 \text{ in.}) + 5(0.525 \text{ in.}) \]
\[ = 4.25 \text{ in.} \]

From AISC Specification Section K1.6:

\[ 5t_p + N = 5(0.500 \text{ in.}) + 4.25 \text{ in.} \]
\[ = 6.75 \text{ in.} < B = 8.00 \text{ in.} \]
Because \((5t_p + N) < B\), the available strength of the HSS is normally computed by summing the contributions of the two walls into which the load is distributed. However, in this example, only one wall is subjected to the compression load, \(C_r\).

**HSS wall local yielding**

Check the limit state of HSS wall local yielding using the following equation from AISC Specification Section K1.6(i). (Table 7-2 of this Design Guide provides the equation for two walls.) For one wall:

\[
R_n = F_F \left[ 5t_p + N \right] \leq BF t
\]

\[
= 46 \text{ksi} \left(0.233 \text{ in.}\right) \left[5(0.500 \text{ in.}) + 4.25 \text{ in.}\right] \leq 8.00 \text{ in.}(46 \text{ksi})(0.233 \text{ in.})
\]

\[
= 72.3 \text{kips} < 85.7 \text{kips}
\]

The available strength is,

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 1.00 )</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>( \phi R_n = 1.00(72.3 \text{ kips}) )</td>
<td>( R_n/\Omega = \frac{72.3 \text{ kips}}{1.50} )</td>
</tr>
<tr>
<td>( = 72.3 \text{ kips} )</td>
<td>( = 48.2 \text{ kips} )</td>
</tr>
<tr>
<td>72.3 kips &gt; 60.2 kips</td>
<td>( \text{o.k.} )</td>
</tr>
</tbody>
</table>

**HSS wall local crippling**

Check the limit state of HSS wall local crippling using the following equation from AISC Specification Section K1.6(ii) and Table 7-2.

\[
R_n = 0.80t \left[ 1 + 6N \left( \frac{t}{t_p} \right)^{1.5} \right] \sqrt{\frac{EF t_p}{t}}
\]

\[
= 0.80 \left(0.233 \text{ in.} \right)^2 \left[ 1 + 6 \left(\frac{4.25 \text{ in.}}{8.00 \text{ in.}} \right) \left(\frac{0.233 \text{ in.}}{0.500 \text{ in.}} \right)^{1.5} \right] \sqrt{(29,000 \text{ ksi})(46 \text{ ksi})(0.500 \text{ in.})} \frac{1}{0.233 \text{ in.}}
\]

\[
= 148 \text{kips}
\]

The available strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.75(148 \text{ kips}) )</td>
<td>( R_n/\Omega = \frac{148 \text{ kips}}{2.00} )</td>
</tr>
<tr>
<td>( = 111 \text{ kips} )</td>
<td>( = 74.0 \text{ kips} )</td>
</tr>
<tr>
<td>111 kips &gt; 60.2 kips</td>
<td>( \text{o.k.} )</td>
</tr>
<tr>
<td></td>
<td>74.0 kips &gt; 40.1 kips</td>
</tr>
</tbody>
</table>

**Weld size between the HSS and cap plate**

Assuming a 45° maximum load dispersion angle, the weld of the cap plate to the HSS can be checked by evaluating a load per inch of the lesser of \(T/g\) or \(T/(d_b + 2b)\), where \(b\) is the distance from the bolt center to the face of the HSS. If \(2g\) or \(2(d_b + 2b)\) exceeds the width of the HSS, \(B\), the weld should be checked for \(2T/B\).

Check if \(2g \leq B\) and \(2(d_b + 2b) \leq B\):

\[
2g \leq B
\]

\[
2(3.50 \text{ in.}) = 7.00 \text{ in.} < 8.00 \text{ in.} \quad \text{o.k.}
\]

\[
2(d_b + 2b) \leq B
\]

\[
2 \left[ \frac{3}{8} \text{ in.} + 2(11.0 \text{ in.} - 8.00 \text{ in.})/2 \right] = 7.50 \text{ in.} < 8.00 \text{ in.} \quad \text{o.k.}
\]
Determine the greater of $T_u/g$ or $T_a/(d_u + 2b)$:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>With $T_e = T_u$</td>
<td>With $T_e = T_u$</td>
</tr>
<tr>
<td>$T_u/g = 7.60 \text{ kips} \div 3.50 \text{ in.}$</td>
<td>$T_a/g = 5.05 \text{ kips} \div 3.50 \text{ in.}$</td>
</tr>
<tr>
<td>= 2.17 kips/in.</td>
<td>= 1.44 kips/in.</td>
</tr>
<tr>
<td>$T_u/(d_u + 2b) = 7.60 \text{ kips} \div (\frac{1}{8} \text{ in.} + 2(11.0 \text{ in.} - 8.00 \text{ in.})$</td>
<td>$T_a/(d_u + 2b) = 5.05 \text{ kips} \div (\frac{1}{8} \text{ in.} + 2(11.0 \text{ in.} - 8.00 \text{ in.})$</td>
</tr>
<tr>
<td>= 1.13 kips/in.</td>
<td>= 0.748 kips/in.</td>
</tr>
<tr>
<td>Use $T_u/g$ in the weld size determination.</td>
<td>Use $T_a/(d_u + 2b)$ in the weld size determination.</td>
</tr>
</tbody>
</table>

Using the procedure given in the AISC Manual Part 8, determine the weld size required.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_n = 1.392Dl$</td>
<td>$R_a/\Omega = 0.928Dl$</td>
</tr>
<tr>
<td>$\phi R_a/l \geq T_u/g$</td>
<td>$R_a/\Omega l \geq T_u/g$</td>
</tr>
<tr>
<td>$1.392D \geq 7.60 \text{ kips} \div 3.50 \text{ in.}$</td>
<td>$0.928D \geq 5.05 \text{ kips} \div 3.50 \text{ in.}$</td>
</tr>
<tr>
<td>$D \geq 1.56$ sixteenths-of-an-inch</td>
<td>$D \geq 1.55$ sixteenths-of-an-inch</td>
</tr>
<tr>
<td>Use $\frac{1}{8}$-in. minimum weld size with 70-ksi filler metal.</td>
<td>Use $\frac{1}{8}$-in. minimum weld size with 70-ksi filler metal.</td>
</tr>
</tbody>
</table>

**Example 4.2—Through-Plate Connection**

**Given:**

Design a through-plate connection as shown in Figure 4-4. Use 1-in.-diameter A325-N bolts in standard holes and 70-ksi filler metal. The applied dead and live loads are shown in Figure 4-4. The loads on the two beams can be reversed; therefore, the connection should be symmetric.

![Diagram of Through-Plate Connection](image)

**Fig. 4-4. Through-plate moment connection.**
From *AISC Manual* Tables 2-3 and 2-4, the material properties are as follows:

**HSS20×12×½**
- ASTM A500 Grade B
- \( F_y = 46 \text{ ksi} \)
- \( F_u = 58 \text{ ksi} \)

**Plates**
- ASTM A36
- \( F_{yp} = 36 \text{ ksi} \)
- \( F_{up} = 58 \text{ ksi} \)

**W16×57**
- ASTM A992
- \( F_y = 50 \text{ ksi} \)
- \( F_u = 65 \text{ ksi} \)

From *AISC Manual* Table 1-11, the HSS geometric properties are as follows:

**HSS20×12×½**
- \( B = 12.0 \text{ in.} \)
- \( H = 20.0 \text{ in.} \)
- \( t = 0.465 \text{ in.} \)

From *AISC Manual* Table 1-1, the W-shape geometric properties are as follows:

**W16×57**
- \( b_f = 7.12 \text{ in.} \)
- \( d = 16.4 \text{ in.} \)
- \( t_f = 0.715 \text{ in.} \)
- \( g = 3\frac{1}{2} \text{ in.} \)

**Solution:**

From Chapter 2 of ASCE 7, the required strengths are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{u,\text{left}} = 1.2(8.00 \text{ kips}) )</td>
<td>( V_{u,\text{left}} = 8.00 \text{ kips} )</td>
</tr>
<tr>
<td>= 9.60 kips</td>
<td></td>
</tr>
<tr>
<td>( V_{u,\text{right}} = 1.2(8.00 \text{ kips}) + 1.6(24.0 \text{ kips}) )</td>
<td>( V_{u,\text{right}} = 8.00 \text{ kips} + 24.0 \text{ kips} )</td>
</tr>
<tr>
<td>= 48.0 kips</td>
<td>= 32.0 kips</td>
</tr>
<tr>
<td>( M_{u,\text{left}} = 1.2(60.0 \text{ kip-ft}) )</td>
<td>( M_{u,\text{left}} = 60.0 \text{ kip-ft} )</td>
</tr>
<tr>
<td>= 72.0 kip-ft</td>
<td></td>
</tr>
<tr>
<td>( M_{u,\text{right}} = 1.2(60.0 \text{ kip-ft}) + 1.6(180 \text{ kip-ft}) )</td>
<td>( M_{u,\text{right}} = 60.0 \text{ kip-ft} + 180 \text{ kip-ft} )</td>
</tr>
<tr>
<td>= 360 kip-ft</td>
<td>= 240 kip-ft</td>
</tr>
<tr>
<td>( P_{u,\text{top}} = 1.2(48.0 \text{ kips}) + 1.6(144 \text{ kips}) )</td>
<td>( P_{u,\text{top}} = 48.0 \text{ kips} + 144 \text{ kips} )</td>
</tr>
<tr>
<td>= 288 kips</td>
<td>= 192 kips</td>
</tr>
</tbody>
</table>
The required axial and flexural strength for connection equilibrium is determined as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum F_r = 9.60 \text{ kips} + 48.0 \text{ kips} + 288 \text{ kips} - P_{a\text{conn}}$</td>
<td>$\sum F_r = 8.00 \text{ kips} + 32.0 \text{ kips} + 192 \text{ kips} - P_{a\text{conn}}$</td>
</tr>
<tr>
<td>$= 0$</td>
<td>$= 0$</td>
</tr>
<tr>
<td>$P_{a\text{conn}} = 346 \text{ kips}$</td>
<td>$P_{a\text{conn}} = 232 \text{ kips}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum M_A = 360 \text{ kip-ft} - 72.0 \text{ kip-ft} - 2M_{a\text{conn}}$</td>
<td>$\sum M_A = 240 \text{ kip-ft} - 60.0 \text{ kip-ft} - 2M_{a\text{conn}}$</td>
</tr>
<tr>
<td>$+ \left(\frac{20.0 \text{ in.}}{2}\right) 48.0 \text{ kips} - 9.60 \text{ kips}$</td>
<td>$+ \left(\frac{20.0 \text{ in.}}{2}\right) 32.0 \text{ kips} - 8.00 \text{ kips}$</td>
</tr>
<tr>
<td>$= 0$</td>
<td>$= 0$</td>
</tr>
<tr>
<td>$M_{a\text{conn}} = 160 \text{ kip-ft}$</td>
<td>$M_{a\text{conn}} = 100 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

Use a plate width of 14.0 in. to permit fillet welding to the HSS.

To begin, use a plate thickness equal to the beam flange thickness ($t_p = t_f$) to determine the approximate maximum force in the plate, $R_u$ and $R_a$.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(d + t_f) R_u = M_{u\text{right}}$</td>
<td>$(d + t_f) R_u = M_{u\text{right}}$</td>
</tr>
<tr>
<td>$(16.4 \text{ in.} + 0.715 \text{ in.}) R_u = 360 \text{ kip-ft} \left(\frac{12 \text{ in.}}{\text{ft}}\right)$</td>
<td>$(16.4 \text{ in.} + 0.715 \text{ in.}) R_u = 240 \text{ kip-ft} \left(\frac{12 \text{ in.}}{\text{ft}}\right)$</td>
</tr>
<tr>
<td>$R_u = 252 \text{ kips}$</td>
<td>$R_u = 168 \text{ kips}$</td>
</tr>
</tbody>
</table>

**Tensile yielding of through-plate**

Using AISC Specification Section J4.1(a), determine the through-plate thickness, $t_p$, based on tensile yielding. The force in the plate, $R_u$ and $R_a$, can then be determined.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.90$</td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$A_s = \frac{R_u}{\phi F_y}$</td>
<td>$A_s = \frac{R_u}{F_y/\Omega}$</td>
</tr>
<tr>
<td>$= 252 \text{ kips}$</td>
<td>$= 168 \text{ kips}$</td>
</tr>
<tr>
<td>$0.90 \left(36 \text{ ksi}\right)$</td>
<td>$36 \text{ ksi}/1.67$</td>
</tr>
<tr>
<td>$= 7.78 \text{ in.}^2$</td>
<td>$= 7.79 \text{ in.}^2$</td>
</tr>
<tr>
<td>$t_p = \frac{7.78 \text{ in.}^2}{14.0 \text{ in.}}$</td>
<td>$t_p = \frac{7.79 \text{ in.}^2}{14.0 \text{ in.}}$</td>
</tr>
<tr>
<td>$= 0.556 \text{ in.}$</td>
<td>$= 0.556 \text{ in.}$</td>
</tr>
<tr>
<td>Use $t_p = % \text{ in.}$</td>
<td>Use $t_p = % \text{ in.}$</td>
</tr>
<tr>
<td>$(d + t_p) R_u = M_{u\text{right}}$</td>
<td>$(d + t_p) R_u = M_{u\text{right}}$</td>
</tr>
<tr>
<td>$(16.4 \text{ in.} + % \text{ in.}) R_u = 360 \text{ kip-ft} \left(\frac{12 \text{ in.}}{\text{ft}}\right)$</td>
<td>$(16.4 \text{ in.} + % \text{ in.}) R_u = 240 \text{ kip-ft} \left(\frac{12 \text{ in.}}{\text{ft}}\right)$</td>
</tr>
<tr>
<td>$R_u = 254 \text{ kips}$</td>
<td>$R_u = 169 \text{ kips}$</td>
</tr>
</tbody>
</table>
Tensile rupture of through-plate

Using the plate thickness determined above, check the limit state of tensile rupture of the through plate. Note that the plate is treated as a splice plate. From AISC Specification Section J4.1(b), the available strength for this limit state can be determined as follows:

\[ R_e = F_e A_e \]  
\[ F_e = F_{up} \]
\[ = 58 \text{ ksi} \]
\[ A_e = A_s - 2 \left( d_e + \frac{1}{16} \text{ in.} \right) t_p \]
\[ = 8.75 \text{ in.}^2 - 2 \left( 1/16 \text{ in.} + 1/16 \text{ in.} \right) \left( 0.625 \text{ in.} \right) \]
\[ = 7.34 \text{ in.}^2 \]
\[ A_e = A_s \leq 0.85 A_s \]
\[ = 0.85 \left( 8.75 \text{ in.}^2 \right) \]
\[ = 7.44 \text{ in.}^2 \]

Therefore, use \( A_e = 7.34 \text{ in.}^2 \)

\[ R_e = 58 \text{ ksi} \left( 7.34 \text{ in.}^2 \right) \]
\[ = 426 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi ) = 0.75</td>
<td>( \Omega ) = 2.00</td>
</tr>
<tr>
<td>( \phi R_e = 0.75 \left( 426 \text{ kips} \right) )</td>
<td>( \frac{R_e}{\Omega} = \frac{426 \text{ kips}}{2.00} )</td>
</tr>
<tr>
<td>= 320 \text{ kips}</td>
<td>= 213 \text{ kips}</td>
</tr>
<tr>
<td>320 \text{ kips} &gt; 254 \text{ kips} o.k.</td>
<td>213 \text{ kips} &gt; 169 \text{ kips} o.k.</td>
</tr>
</tbody>
</table>

Compressive yielding and buckling of through-plate

Using AISC Manual Table 17-27, calculate the radius of gyration of the plate:

\[ r = \frac{t_p}{\sqrt{12}} \]
\[ = \frac{5/8 \text{ in.}}{\sqrt{12}} \]
\[ = 0.180 \text{ in.} \]

Determine the equation for the nominal compressive strength, \( P_n \), from AISC Specification Section J4.4. Use a bolt spacing, \( s \), of 3.00 in. for \( L \) and a distance of 3.00 in. from the first bolt to the face of the HSS.

\[ KL/r = \frac{1.00 \left( 3.00 \text{ in.} \right)}{0.180} \]
\[ = 16.7 < 25 \]

Therefore, use \( P_n = F_s A_g \)

This check is the same as the tension yield check and is o.k. Use a PL5/8×14 for the through-plate.
Bolt shear rupture
From AISC Manual Table 7-1, the available shear strength per bolt, based on the limit state of shear rupture, is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi, r_n = 28.3 \text{ kips}$</td>
<td>$\frac{R_n}{\Omega} = 18.8 \text{ kips}$</td>
</tr>
<tr>
<td>The required number of bolts, $n$, is:</td>
<td>The required number of bolts, $n$, is:</td>
</tr>
<tr>
<td>$n = \frac{R_n}{\phi, r_n}$</td>
<td>$n = \frac{R_n}{r_n/\Omega}$</td>
</tr>
<tr>
<td>$= \frac{254 \text{ kips}}{28.3 \text{ kips}}$</td>
<td>$= \frac{169 \text{ kips}}{18.8 \text{ kips}}$</td>
</tr>
<tr>
<td>$= 8.98$</td>
<td>$= 8.99$</td>
</tr>
</tbody>
</table>

Try two rows of five bolts on each side of the HSS with 3.00-in. spacing, 3.00 in. from the HSS to the first bolt, and 1/4-in. edge distance per AISC Specification Section J3.4.

Bearing on the plate
Use the following equation from AISC Specification Section J3.10(a)(i), when deformation at the bolt hole at service load is a design consideration.

$$ R_n = 1.2L_t F_d \leq 2.4dt F_u $$  
$$ F_u = F_{up} $$  
$$ t = t_p $$  
$$ = 58 \text{ ksi} $$  
$$ \Omega = 0.625 \text{ in.} $$

For the end bolts
$$ L_t = 1\frac{1}{4} \text{ in.} - 1/16 \text{ in.}/2 $$  
$$ = 1.22 \text{ in.} $$

$$ R_n = 1.2(1.22 \text{ in.})(\frac{3}{8} \text{ in.})(58 \text{ ksi}) $$
$$ = 53.1 \text{ kips} < 2.4(1.00 \text{ in.})(\frac{3}{8} \text{ in.})(58 \text{ ksi}) $$
$$ < 87.0 \text{ kips} $$

Use $R_n = 53.1 \text{ kips}$.

The available bearing strength is determined as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>For the end bolts</td>
<td>For the end bolts</td>
</tr>
<tr>
<td>$\phi, r_n = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi, R_n = 75(53.1 \text{ kips})$</td>
<td>$R_n = \frac{53.1 \text{ kips}}{2.00}$</td>
</tr>
<tr>
<td>$= 39.8 \text{ kips}$</td>
<td>$= 26.6 \text{ kips}$</td>
</tr>
</tbody>
</table>

From AISC Manual Table 7-5, the available bearing strength based on 3-in. bolt spacing is:

For the interior bolts

$$ \phi, r_n = 101 \text{ kips per inch of thickness} $$

$$ \phi R_n = 101 \text{ kips/in. (3/8 in.)} $$

$$ = 63.1 \text{ kips} $$

$$ \Omega = 101 \text{ kips per inch of thickness} $$

$$ \frac{R_n}{\Omega} = 67.4 \text{ kips/in. (3/8 in.)} $$

$$ = 42.1 \text{ kips} $$
For the 10 bolts
\[ \phi R_n = 2\left(39.8 \text{ kips}\right) + 8\left(63.1 \text{ kips}\right) \]
\[ = 584 \text{ kips} \]
\[ 584 \text{ kips} > 254 \text{ kips} \quad \text{o.k.} \]

For the 10 bolts
\[ \frac{R_n}{\Omega} = 2\left(26.6 \text{ kips}\right) + 8\left(42.1 \text{ kips}\right) \]
\[ = 390 \text{ kips} \]
\[ 390 \text{ kips} > 169 \text{ kips} \quad \text{o.k.} \]

**Bearing on the beam flange**

Use the following equation from AISC Specification Section J3.10(a)(i), when deformation at the bolt hole at service load is a design consideration.

\[ R_n = 1.2L_t F_d \leq 2.4d t F_u \]
\[ F_u = 65 \text{ ksi} \]
\[ t = t_f \]
\[ = 0.715 \text{ in.} \]

For the end bolts, use a 2-in. edge distance at the end of the beam.
\[ L_e = 2 \text{ in.} - 1/16 \text{ in.}/2 \]
\[ = 1.47 \text{ in.} \]
\[ R_n = 1.2\left(1.47 \text{ in.}\right)\left(0.715 \text{ in.}\right)\left(65 \text{ ksi}\right) \]
\[ = 82.0 \text{ kips} < 2.4\left(1.00 \text{ in.}\right)\left(0.715 \text{ in.}\right)\left(65 \text{ ksi}\right) \]
\[ < 111 \text{ kips} \]

Use \( R_n = 82.0 \text{ kips} \).

<table>
<thead>
<tr>
<th>LRFD (cont.)</th>
<th>ASD (cont.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>For the 10 bolts</td>
<td>For the 10 bolts</td>
</tr>
<tr>
<td>[ \phi R_n = 2\left(39.8 \text{ kips}\right) + 8\left(63.1 \text{ kips}\right) ]</td>
<td>[ \frac{R_n}{\Omega} = 2\left(26.6 \text{ kips}\right) + 8\left(42.1 \text{ kips}\right) ]</td>
</tr>
<tr>
<td>[ = 584 \text{ kips} ]</td>
<td>[ = 390 \text{ kips} ]</td>
</tr>
<tr>
<td>584 kips &gt; 254 kips \quad \text{o.k.}</td>
<td>390 kips &gt; 169 kips \quad \text{o.k.}</td>
</tr>
</tbody>
</table>
Bearing does not control.
Length of through plate = 2\left[1\frac{1}{4} \text{ in.} + 5(3.00 \text{ in.})\right] + 20.0 \text{ in.}
= 53.5 \text{ in.}

Block shear rupture of the through-plate

The controlling failure surface for the block shear rupture limit state is the shear area through the two lines of bolts and the tensile area between the two bolts closest to the HSS. From AISC Specification Section J4.3, the available block shear rupture strength is determined as follows:

\[ R_n = 0.6F_uA_{uy} + U_{bs}F_uA_{uy} \leq 0.6F_yA_{yu} + U_{bs}F_uA_{uy} \]  \hspace{1cm} (Spec. Eq. J4-5)

\[ F_u = F_{uy} \]
\[ = 58 \text{ ksi} \]
\[ F_y = F_{yu} \]
\[ = 36 \text{ ksi} \]
\[ L_{uy} = 1\frac{1}{4} \text{ in.} + 4(3.00 \text{ in.}) \]
\[ = 13.8 \text{ in.} \]
\[ A_{yu} = 2(13.8 \text{ in.})(\frac{5}{8} \text{ in.}) \]
\[ = 17.3 \text{ in.}^2 \]
\[ A_{yu} = 17.3 \text{ in.}^2 - 2(4.5)(1\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.})(\frac{5}{8} \text{ in.}) \]
\[ = 11.0 \text{ in.}^2 \]
\[ g = 3\frac{1}{2} \text{ in.} \]
\[ A_{uy} = \frac{5}{8} \text{ in.}[3\frac{1}{2} \text{ in.} - (1\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.})] \]
\[ = 1.48 \text{ in.}^2 \]
\[ U_{bs} = 1 \text{ since tension is uniform} \]

The left side of the inequality given in Equation J4-5 is:
\[ 0.6F_uA_{uy} + U_{bs}F_uA_{uy} = 0.6(58 \text{ ksi})(11.0 \text{ in.}^2) + 1(58 \text{ ksi})(1.48 \text{ in.}^2) \]
\[ = 469 \text{ kips} \]

The right side of the inequality given in Equation J4-5 is:
\[ 0.6F_yA_{yu} + U_{bs}F_yA_{yu} = 0.6(36 \text{ ksi})(17.3 \text{ in.}^2) + 1(58 \text{ ksi})(1.48 \text{ in.}^2) \]
\[ = 460 \text{ kips} \]

Because 469 > 460 kips, use \( R_u = 460 \) kips.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi = 0.75 ]</td>
<td>[ \Omega = 2.00 ]</td>
</tr>
<tr>
<td>[ \phi R_u = 0.75(460 \text{ kips}) ] = 345 kips</td>
<td>[ \frac{R_u}{\Omega} = \frac{460 \text{ kips}}{2.00} ] = 230 kips</td>
</tr>
<tr>
<td>345 kips &gt; 254 kips \text{ o.k.}</td>
<td>230 kips &gt; 169 kips \text{ o.k.}</td>
</tr>
</tbody>
</table>
Block shear rupture of the beam flange.

The block shear pattern is shear through the lines of bolts and tension from the last bolt to the tips of the flange on both sides. From AISC Specification Section J4.3, the available block shear rupture strength is determined as follows:

\[
R_n = 0.6F_uA_{n} + U_{n}F_yA_{nt} \leq 0.6F_yA_{nt} + U_{n}F_uA_{n} \quad \text{(Spec. Eq. J4-5)}
\]

- \(F_u = 65 \text{ ksi}\)
- \(F_y = 50 \text{ ksi}\)
- \(L_{sv} = 2\text{ in.} + 4(3.00 \text{ in.}) = 14.0 \text{ in.}\)
- \(A_{sv} = 2(14.0 \text{ in.})(0.715 \text{ in.}) = 20.0 \text{ in.}^2\)
- \(A_{nt} = 20.0 \text{ in.}^2 - 2(4.5)(1/16 \text{ in.} + 1/16 \text{ in.})(0.715 \text{ in.}) = 12.8 \text{ in.}^2\)
- \(g = 3/2 \text{ in.}\)
- \(A_{n} = 0.715 \text{ in.}[7.12 \text{ in.} - 3/2 \text{ in.} - (1/16 \text{ in.} + 1/16 \text{ in.})] = 1.78 \text{ in.}^2\)
- \(U_{n} = 1\) since tension is uniform

The left side of the inequality given in Equation J4-5 is:

\[
0.6F_uA_{n} + U_{n}F_yA_{nt} = 0.6(65 \text{ ksi})(12.8 \text{ in.}^2) + 1(65 \text{ ksi})(1.78 \text{ in.}^2) = 615 \text{ kips}
\]

The right side of the inequality given in Equation J4-5 is:

\[
0.6F_yA_{nt} + U_{n}F_uA_{n} = 0.6(50 \text{ ksi})(20.0 \text{ in.}^2) + 1(65 \text{ ksi})(1.78 \text{ in.}^2) = 716 \text{ kips}
\]

Because 615 < 716 kips, use \(R_n = 615 \text{ kips}\).

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi = 0.75)</td>
<td>(\Omega = 2.00)</td>
</tr>
<tr>
<td>(\phi R_n = 0.75(615 \text{ kips}))</td>
<td>(\frac{R_n}{\Omega} = \frac{615 \text{ kips}}{2.00} = 308 \text{ kips})</td>
</tr>
<tr>
<td>= 461 kips</td>
<td>= 308 kips</td>
</tr>
<tr>
<td>461 kips &gt; 254 kips o.k.</td>
<td>308 kips &gt; 169 kips o.k.</td>
</tr>
</tbody>
</table>

Weld size for plate-to-HSS connection

Design the plate-to-HSS weld for combined axial force and moment on the compression side where they are additive.

Referring to Figure 4-5:

\[
A_{nt} = 2(B + H)I_{thrust}
= 2(12.0 \text{ in.} + 20.0 \text{ in.})I_{thrust}
= 64.0I_{thrust} \text{ in.}^2
\]
\[ I = 2 \left( \frac{H^2 t_{\text{throat}}}{12} \right) + 2B t_{\text{throat}} \left( \frac{H}{2} \right)^2 \]
\[ = 2 \left[ \frac{(20.0 \text{ in.})^3}{12} t_{\text{throat}} \right] + 2(12.0 \text{ in.}) t_{\text{throat}} \left( \frac{20.0 \text{ in.}}{2} \right)^2 \]
\[ = 3,730 t_{\text{throat}} \text{ in.}^4 \]

Note that the throat thickness is neglected for simplicity in the squared portion \((H/2)^2\) of the second term in the calculation of \(I\) above, as its effect would be negligible.

Required resistance per unit length of weld
\[ = \frac{P_{\text{conn}}}{A_e / t_{\text{throat}}} + \frac{M_{\text{conn}} (H/2)}{I / t_{\text{throat}}} \]

Use \(F_{\text{xx}} = 70 \text{ ksi}\).

Using the procedure provided in Part 8 of the AISC Manual, determine the weld size required to connect the plate to the HSS.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_e = 1.392 D I )</td>
<td>( R_e / \Omega = 0.928 D I )</td>
</tr>
<tr>
<td>( \phi R_e / I = 1.392 D \geq \frac{P_{\text{conn}}}{A_e / t_{\text{throat}}} + \frac{M_{\text{conn}} (H/2)}{I / t_{\text{throat}}} )</td>
<td>( R_e / \Omega = 0.928 D \geq \frac{P_{\text{conn}}}{A_e / t_{\text{throat}}} + \frac{M_{\text{conn}} (H/2)}{I / t_{\text{throat}}} )</td>
</tr>
<tr>
<td>( 1.392 D \geq 346 \text{ kips} )</td>
<td>( 0.928 D \geq 232 \text{ kips} )</td>
</tr>
<tr>
<td>( \frac{64.0 \text{ in.}}{2} )</td>
<td>( \frac{64.0 \text{ in.}}{2} )</td>
</tr>
<tr>
<td>( + \frac{160 \text{ kip-ft}(12 \text{ in./ft})(20.0 \text{ in./2})}{3,730 \text{ in.}^4} )</td>
<td>( + \frac{100 \text{ kip-ft}(12 \text{ in./ft})(20.0 \text{ in./2})}{3,730 \text{ in.}^4} )</td>
</tr>
<tr>
<td>( 1.392 D \geq 10.6 \text{ kips/in.} )</td>
<td>( 0.928 D \geq 6.84 \text{ kips/in.} )</td>
</tr>
<tr>
<td>( D \geq 7.61 \text{ sixteenths-of-an-inch} )</td>
<td>( D \geq 7.37 \text{ sixteenths-of-an-inch} )</td>
</tr>
<tr>
<td>Use (1/2)-in. weld</td>
<td>Use (1/2)-in. weld</td>
</tr>
</tbody>
</table>

Available shear strength of the single plate connection

Note that the controlling loads (see beginning of this example) are \(V_{u \text{right}} = 48.0 \text{ kips}\) and \(V_{u \text{right}} = 32.0 \text{ kips}\).

Using AISC Manual Table 10-9a with three 1-in.-diameter ASTM A325-N bolts and \(3/8\)-in. plate, 9\(1/2\) in. long, and \(F_{yp} = 36 \text{ ksi}\), determine the available shear strength of the single plate.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_e = 59.9 \text{ kips} )</td>
<td>( R_e / \Omega = 40.0 \text{ kips} )</td>
</tr>
<tr>
<td>( 59.9 \text{ kips} &gt; 48.0 \text{ kips} )</td>
<td>( 40.0 \text{ kips} &gt; 32.0 \text{ kips} )</td>
</tr>
</tbody>
</table>

Available shear strength of the single plate connection

Note that the controlling loads (see beginning of this example) are \(V_{u \text{right}} = 48.0 \text{ kips}\) and \(V_{u \text{right}} = 32.0 \text{ kips}\).

Using AISC Manual Table 10-9a with three 1-in.-diameter ASTM A325-N bolts and \(3/8\)-in. plate, 9\(1/2\) in. long, and \(F_{yp} = 36 \text{ ksi}\), determine the available shear strength of the single plate.

- **LRFD**
  - \( \phi R_e = 59.9 \text{ kips} \)
  - \( 59.9 \text{ kips} > 48.0 \text{ kips} \) o.k.

- **ASD**
  - \( R_e / \Omega = 40.0 \text{ kips} \)
  - \( 40.0 \text{ kips} > 32.0 \text{ kips} \) o.k.

![Fig. 4-5. Weld lengths for plate-to-HSS connection.](image)
From AISC Manual Table 10-9a, the weld size is \( \frac{1}{4} \) in.

Use a plate width of 3 in. from the HSS to the center of the bolt hole + 1\( \frac{1}{4} \)-in. horizontal edge distance per AISC Specification Table J3.4 and AISC Manual Part 10.

Therefore

\[
\text{plate width} = 3.00 \text{ in.} + 1\frac{1}{4} \text{ in.} = 4.75 \text{ in.}
\]

The length of the HSS segment between the through plates must be longer than the nominal beam depth to account for mill tolerances. AISC Manual Table 1-22 states that for a section of nominal depth greater than 12 in., the maximum depth over the theoretical depth at any cross-section is \( \frac{1}{8} \) in. Therefore, the length of the HSS segment should be greater than or equal to 16.4 in. + 0.125 in. = 16.525 in. During erection, any space between the through plates and the beam flanges should be filled with conventional shims or finger shims.

**Example 4.3—Directly Welded Connection**

**Given:**

Determine the moment transfer capacity between a W16×57 beam and an HSS10×10×\( \frac{1}{2} \) in the configuration shown in Figure 4-2. The HSS column carries axial dead and live loads of \( P_D = 100 \) kips and \( P_L = 300 \) kips, respectively.

From AISC Manual Table 2-3, the material properties are as follows:

- HSS10×10×\( \frac{1}{2} \)
  - ASTM A500 Grade B
  - \( F_y = 46 \) ksi
  - \( F_u = 58 \) ksi
- W16×57
  - ASTM A992
  - \( F_y = 50 \) ksi
  - \( F_u = 65 \) ksi

From AISC Manual Table 1-12, the HSS geometric properties are as follows:

- HSS10×10×\( \frac{1}{2} \)
  - \( A = 17.2 \) in.\(^2\)
  - \( B = 10.0 \) in.
  - \( H = 10.0 \) in.
  - \( t = 0.465 \) in.

From AISC Manual Table 1-1, the W-shape geometric properties are as follows:

- W16×57
  - \( b_f = 7.12 \) in.
  - \( d = 16.4 \) in.
  - \( t_f = 0.715 \) in.

**Solution:**

*Fit on the flat of the HSS wall*

Determine whether there is adequate width to weld the W-shape to the HSS wall:

\[
\begin{align*}
  b_f &= 7.12 \text{ in.} < B - 3t \\
  B - 3t &= 10.0 \text{ in.} - 3(0.465 \text{ in.}) = 8.61 \text{ in.} \\
  7.12 \text{ in.} &< 8.61 \text{ in.} \quad \text{o.k.}
\end{align*}
\]
In applying AISC Specification Section K1.3b for plates for use with the beam flange, the variables $B_p = b_f$, $t_p = t_f$, and $F_{yp}$, refer to the beam.

**Limits of applicability**

The following limits of applicability given in AISC Specification Section K1.2 and Table 7-2A of this Design Guide must be met in order to apply the criteria given in AISC Specification Section K1.3.

\[
\begin{align*}
\theta &= 90^\circ \geq 30^\circ \quad \text{o.k.} \\
0.25 \leq b_f/B &= 0.712 \leq 1.0 \quad \text{o.k.} \\
B/t &= 21.5 \leq 35 \quad \text{o.k.} \\
F_y &= 46 \text{ ksi} \leq 52 \text{ ksi} \quad \text{o.k.} \\
F_{yp} &= 50 \text{ ksi} \leq 52 \text{ ksi} \quad \text{o.k.} \\
F_i/F_y &= 0.793 \leq 0.8 \quad \text{o.k.} \\
F_{yp}/F_{y'p} &= 0.769 \leq 0.8 \quad \text{o.k.}
\end{align*}
\]

**Local yielding of the beam flange due to uneven load distribution**

Using AISC Specification Section K1.3b(a), determine the available strength based on the limit state of local yielding due to uneven load distribution in the beam flange.

\[
R_n = \frac{10}{B/t} F_t B_p \leq F_{y'p} t_p B_p
\]

\[
R_n = \frac{10}{21.5} \left(46 \text{ ksi}\right) \left(0.465 \text{ in.}\right) \left(7.12 \text{ in.}\right) \leq 50 \text{ ksi} \left(0.715 \text{ in.}\right) \left(7.12 \text{ in.}\right)
\]

\[
= 70.8 \text{ kips} < 255 \text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.95$</td>
<td>$\Omega = 1.58$</td>
</tr>
<tr>
<td>$\phi R_n = 0.95 \left(70.8 \text{ kips}\right)$</td>
<td>$R_n / \Omega = \frac{70.8 \text{ kips}}{1.58} = 44.8 \text{ kips}$</td>
</tr>
</tbody>
</table>

**HSS shear yielding (punching)**

Using AISC Specification Section K1.3b(b), determine whether the limit state of shear yielding (punching) of the HSS is applicable.

Because $B_p = b_f = 7.12 \text{ in.} < 0.85B = 0.85(10 \text{ in.}) = 8.50 \text{ in.}$, this limit state need not be checked.

**Sidewall strength**

Using AISC Specification Section K1.3b(c), determine whether the limit state of sidewall compression loading is applicable.

Because $b_f/B = \beta = 0.712 \neq 1.0$, this limit state need not be checked.
Available flexural strength

Determine the available moment transfer as the flange force, $F_f$, times the distance between the flange centers.

$$M = F_f \left( d - t_f \right)$$
$$= 15.7F_f \text{ kip-in.}$$

The available flexural strength is determined as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_f = \phi R_s = 67.3 \text{ kips}$</td>
<td>$F_f = R_s/\Omega = 44.8 \text{ kips}$</td>
</tr>
<tr>
<td>$\phi M_n = \frac{15.7 \text{ in.}}{12 \text{ in./ft}} \left( 67.3 \text{ kips} \right)$</td>
<td>$M_n/\Omega = \frac{15.7 \text{ in.}}{12 \text{ in./ft}} \left( 44.8 \text{ kips} \right)$</td>
</tr>
<tr>
<td>= 88.1 kip-ft</td>
<td>= 58.6 kip-ft</td>
</tr>
</tbody>
</table>

Note: From AISC Manual Table 3-2
W16×57 $\phi M_n = 394 \text{ kip-ft}$
From AISC Manual Table 3-13
HSS10×10×½ $\phi M_n = 210 \text{ kip-ft}$

Note: From AISC Manual Table 3-2
W16×57 $M_n/\Omega = 262 \text{ kip-ft}$
From AISC Manual Table 3-13
HSS10×10×½ $M_n/\Omega = 139 \text{ kip-ft}$

This example illustrates the limited flexural strength that can be achieved for W-shape beams directly welded to HSS columns. Another example of a W-shape beam directly welded to a round HSS column is given in Chapter 7 of this Design Guide (Example 7.2).
Chapter 5
Tension and Compression Connections

5.1 TYPES OF END CONNECTIONS

Round, rectangular and square HSS are frequently used as bracing members in steel construction. These members may be subject to tension or compression loads, and the end connections to supporting elements of the structure must be capable of resisting the required forces. In many applications, the forces in the member are much less than the yield capacity of the HSS. For example, the resistance of a slender compression brace is less than its yield strength. If the same force is developed in the reversed load condition, the tension force will also be considerably less than the yield capacity.

One type of connection for these members is the end tee bolted connection shown in Figure 5-1(a). The tee can be either a WT section or built up with two plates. The flange is welded to the end of the HSS with a continuous weld around the perimeter, and the stem is bolted to the supporting attachment. A single or multiple lines of bolts may be used. The transfer of force through the bolts to the connecting element is through shear. Another type of end connection is the slotted HSS/gusset plate connection shown in Figure 5-1(b). The gusset is welded to the HSS in the slot, and the projecting portion of the gusset is similar to the stem of the end tee connection. Figure 5-1(c) depicts a typical field-welded HSS to gusset plate connection.

Due to shear lag, the connections shown in Figure 5-1(b) cannot develop the yield strength of the member according to AISC Specification Table D3.1, except for a round slotted HSS (or slotted plate) with the weld length $l \geq 1.3D$.  

Fig. 5-1. Bracing connections with bolts or welds in shear.
5.2 END TEE CONNECTIONS

The length and width of the tee flange must extend beyond the HSS to provide sufficient shelf to deposit the fillet weld around the perimeter of the HSS.

For tension loads, the limit states for the end tee connections are:

1. Strength of the weld connecting the tee flange to the HSS
2. Strength of the weld connecting the stem to the cap plate, for built-up tees
3. Strength of the HSS wall in local yielding
4. Strength of the tee flange for through-thickness shear adjacent to the stem
5. Strength based on bolting to the tee stem
6. Strength of the stem in yielding, rupture and block shear

For compressive loading, two other limit states must be considered:

7. Strength of the HSS wall in crippling, for square/rectangular HSS
8. Strength based on buckling of the tee stem
The procedure for determining the available buckling strength of the tee stem is based on an eccentrically loaded compression member as shown in Figure 5-5. Both ends are assumed fixed and can move laterally, resulting in the effective length factor, $K = 1.2$. The geometric properties over the entire length, $L_e$, are taken as those of the thinner of the tee stem or supporting gusset. It is possible to add a stiffener to enhance the buckling strength; however, it is generally more economical to thicken the tee stem and/or gusset.

The interaction equations from AISC Specification Section H1.1 for combined flexure and compression are:

$$\frac{P_c}{P_r} + \frac{8}{9} \left( \frac{M_d}{M_c} \right) \leq 1.0 \quad \text{for} \quad \frac{P_r}{P_r} \geq 0.2 \quad (5-1)$$

$$\frac{P_r}{P_r} + \frac{M_d}{M_c} \leq 1.0 \quad \text{for} \quad \frac{P_r}{P_r} < 0.2 \quad (5-2)$$

where $P_r$ and $M_c$ are the available strengths in axial compression and flexure. Using $M_c = P_r e/2$ (see Figure 5-5), the equations can be solved for the maximum compression strength and Equations 5-1 and 5-2 become:

$$P_r \leq \frac{1}{4} \left( \frac{P_r}{P_r} + \frac{M_c}{M_c} \right) \quad \text{for} \quad \frac{P_r}{P_r} \geq 0.2 \quad (5-3)$$

$$P_r \leq \frac{2}{e} \left( \frac{P_r}{P_r} + \frac{M_c}{M_c} \right) \quad \text{for} \quad \frac{P_r}{P_r} < 0.2 \quad (5-4)$$

Example 5.1 shows a calculation for an end tee connection.

### 5.3 SLOTTED HSS/GUSSET CONNECTION

The slot in the HSS for the gusset plate is made wider than the plate thickness to facilitate fabrication and erection. Therefore, the weld size must be increased per AWS D1.1 to account for the gap. However, AWS D1.1 permits a $\frac{1}{16}$-in. gap with no increase in weld size. To avoid undercutting of the plate, AISC recommends that the weld not be returned around the end of the plate in the slot. Also, for field-welded braces, the slot length extends beyond the end of the plate to facilitate erection. Consequently, there is a reduced net area at the end of the gusset that, when combined with shear lag, could lead to HSS rupture as a critical limit state. The width of the gusset must extend beyond the HSS to provide a shelf for the fillet welds.

Besides the HSS member limit states, the other limit states that must be considered are:

1. Base metal shear in the HSS
2. Base metal shear in the gusset
3. Shear in the weld connecting the HSS to the gusset
4. Strength based on bolting to the gusset plate

The AISC Specification implies that the length of the welds should be equal to or greater than the distance between the welds, $H$ or $D$, where $H$ is the overall height of a rectangular HSS member and $D$ is the outside dimension of a round HSS, measured in the plane of the connection. The shear lag factors, $U$, in AISC Specification Table D3.1 have no provisions for shorter welds. This is consistent with the requirements for plates where the tension load is transmitted by longitudinal welds only, where no $U$ factor is specified for welds that are shorter than the spacing between them. Example 5.2 shows a calculation for a slotted round/gusset connection.

### 5.4 END PLATE ON ROUNd HSS

The bolts in the end-plate connection shown in Figure 5-4(a) are equally spaced around the circumference. To obtain the minimum end-plate thickness, the distance, $b$, from the HSS wall to the bolt line should be as small as practical, keeping in mind clearances for an impact wrench and minimum spacing requirements, per AISC Specification Section J3.3. The distance, $a$, from the bolt line to the edge of the plate is assumed equal to $b$ in the method used to design the end-plate connection. The limit states are:

1. Yielding of the end plate
2. Tensile strength of the bolts, including prying action
3. Strength of the weld connecting the end plate to the HSS

Due to the complexity of the analysis accounting for prying action and flexure of the plate, it is difficult to directly determine nominal resistances according to the AISC Specification provisions. However, equations are presented (Packer and Henderson, 1997) to determine the thickness of the plate, $t_e$, the number of bolts, $n$, and the weld size, $w$.

$$t_e \geq \frac{2P_r}{\sqrt{F_{wd} \pi f_s}} \quad (5-5)$$

$$n \geq \frac{P_r}{R_b} \left[ 1 - \frac{1}{f_b} + \frac{1}{f_s \ln r_i / r_o} \right] \quad (5-6)$$

$$w \geq \frac{P_r \sqrt{2}}{F_{wd} \pi D} \quad (5-7)$$

![Fig. 5-5. Tee stem/gusset as an eccentrically loaded column.](image)
From AISC Specification Section J2.4, the available weld strength based on the limit state of shear rupture is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{wc} = \phi_c F_w$</td>
<td>$F_{wc} = \frac{F_w}{\Omega_w}$</td>
</tr>
<tr>
<td>where</td>
<td>where</td>
</tr>
<tr>
<td>$F_w = 0.60F_{EXX}$</td>
<td>$F_w = 0.60F_{EXX}$</td>
</tr>
<tr>
<td>$\phi_c = 0.75$</td>
<td>$\Omega_w = 2.00$</td>
</tr>
</tbody>
</table>

5.5 END PLATE ON RECTANGULAR HSS WITH BOLTS ON TWO SIDES

For the design criteria to be valid, the centerline of the bolts in the end-plate connection should not be positioned beyond the corner of the rectangular HSS. The limit states for the end-plate connection bolted on two sides, as in Figure 5-7, are:

1. Yielding of the end plate
2. Tensile strength of the bolts, including prying action
3. Strength of the weld connecting the end plate to the HSS

Due to the complexity of the analysis accounting for prying action and the position of the yield lines in the plate, it is difficult to directly determine nominal resistances according to the AISC Specification provisions. A modified T-stub design procedure can be used to evaluate the connection limit states (Packer and Henderson, 1997). The details of the procedure and background research are presented in the reference.

Plate yielding

$$R_y = \frac{t^2}{2K}[1 + \delta_\alpha]^n$$  (5-14)

Total bolt tension

$$T_n = \frac{P}{n} \left[ 1 + \left( \frac{b'}{a} \right) \left( \frac{\alpha}{1 + \alpha} \right) \right]$$  (5-15)
Weld size

$$w \geq \frac{P\sqrt{2}}{2BcF_w}$$  \hspace{1cm} (5-16)

where

$$K = \frac{4b'}{F_{yp}p}$$  \hspace{1cm} (5-17)

$$a = \text{distance from the bolt centerline to the edge of the fitting, in.}$$

$$a' = a + \frac{d_b}{2}$$  \hspace{1cm} (5-18)

$$a_p = a \leq 1.25b$$  \hspace{1cm} (5-19)

$$b' = b - \frac{d_b}{2} + t$$  \hspace{1cm} (5-20)

$$n = \text{number of bolts}$$

$$a = \frac{K (P_p/n)}{t_p^2} - 1$$  \hspace{1cm} (5-21)

$$\delta = 1 - \frac{d_b}{p}$$  \hspace{1cm} (5-22)

The available tensile strength of a bolt is checked using AISC Manual Table 7-2 as shown below.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c = \phi = 0.90$</td>
<td>$c = \frac{1}{\Omega}$ where $\Omega = 1.67$</td>
</tr>
<tr>
<td>$P_a \leq \phi R_c$</td>
<td>$P_a \leq \frac{R_c}{\Omega}$</td>
</tr>
<tr>
<td>$T_r \leq \phi r_n$</td>
<td>$T_r \leq \frac{r_n}{\Omega}$</td>
</tr>
</tbody>
</table>

From AISC Specification Section J2.4, the available weld strength based on the limit state of shear rupture is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{w0} = \phi_s F_w$</td>
<td>$F_{w0} = \frac{F_w}{\Omega_w}$ where $F_w = 0.60 F_{EXX}$ and $\phi_s = 0.75$</td>
</tr>
</tbody>
</table>

### 5.6 END PLATE ON RECTANGULAR HSS WITH BOLTS ON FOUR SIDES

End-plate connections with bolts on all four sides are more common than with bolts on just two sides. The limit states for the end-plate connection bolted on four sides as shown in Figure 5-8 are:

1. Yielding of the end plate
2. Tensile strength of the bolts, including prying action
3. Strength of the weld connecting the end plate to the HSS

Due to the complexity of the analysis accounting for prying action and the position of yield lines in the plate, it is difficult to directly determine nominal capacities according to the AISC Specification provisions. However, equations are presented to determine the thickness of the plate, $t_p$, the number of bolts, $n$, and the weld size, $w$. The design procedure is based on the AISC design procedure for hanger connections. For the following design criteria to be valid, the centers of the bolt holes should not be positioned beyond the corners of the HSS. The details of the design procedure are presented by Willibald et al. (2003). In the design procedures, the flange-plate material strength is taken as $F_{yp}$, as validated experimentally for HSS end-plate connections by Willibald et al. (2003), rather than the higher capacity $F_{up}$ as in the AISC Manual (pages 9-10 to 9-12).

The following simplified equation from AISC Manual Part 9, gives the minimum plate thickness, $t_{min}$, required for no prying action to occur:

$$t_p \geq t_{44.4} = \sqrt{\frac{4.44 (P_p/n) b'}{p F_{yp}}}$$  \hspace{1cm} (5-23a)

$$t_p \geq t_{66.6} = \sqrt{\frac{6.66 (P_p/n) b'}{p F_{yp}}}$$  \hspace{1cm} (5-23b)

where

- $p$ = length of end plate tributary to each bolt, in.
- $b' = \frac{b}{n}$ = flange height/width divided by the number of bolts to the flange height/width on one side, in. (see Figure 5-8)

![Fig. 5-8. Rectangular end plate with bolts on four sides.](image-url)
[Note that if the HSS is not square, or if the bolting layout is not the same on all four sides, then the bolt pitch used should be the minimum of the bolt pitch for the long and short side (assuming equal values of $a$ and $b$ for the long and short sides); i.e., the bolt pitch is equal to the minimum of $p$ and $p'$ in Figure 5-8 (Willibald et al., 2003)].

The available tensile strength of a bolt is given in AISC Manual Table 7-2 and compared to the required strength as shown below.

From AISC Specification Section J2.4, the weld size, $w$, is determined based on the limit state of shear rupture as follows.

\[
\frac{P_n}{n} \leq \Phi_{n-1} \quad (5-24a)
\]

\[
\frac{P_n}{n} \leq r_n \quad (5-24b)
\]

Additional terms used in the foregoing are defined as follows:

\[
\phi_{n-1} = \frac{1.25b}{a} \quad (5-26a)
\]

\[
\phi_n = \frac{1.25b}{a} \quad (5-26b)
\]

As discussed in AISC Manual Part 9, a lesser required thickness can be determined by designing the connecting element and bolted joint for the effects of prying action. The thickness required to ensure an acceptable combination of fitting strength and stiffness and bolt strength is determined as follows:

\[
t_p \geq \frac{4.44(P_n/n)b'}{pF_{w'p}(1+\delta\alpha)} \quad (5-26a)
\]

\[
t_p \geq \frac{6.66(P_n/n)b'}{pF_{w'p}(1+\delta\alpha')} \quad (5-26b)
\]

The available tensile strength, $\phi_{n-1}$ or $r_n/\Omega$, of the bolt can be determined from AISC Manual Table 7-2, and the following relationship must be checked based on the discussion in Part 9 of the AISC Manual.

\[
\frac{P_n}{n} + \phi_{n-1} \left[ \delta\alpha \left( \frac{t_p}{t_{min}} \right)^2 \right] \leq \phi_n \quad (5-27a)
\]

where

\[t_{min} \text{ is from Equation 5-23a}\]

\[
\alpha = \left[ \frac{P_n \left( \frac{t_{min}}{t_p} \right)^2}{n\phi_{n-1}} - 1 \right] \frac{1}{\delta} \geq 0
\]

From AISC Specification Section J2.4, the weld size, $w$, is determined based on the limit state of shear rupture as follows.

\[
\frac{w}{n} \geq \frac{P_n \sqrt{2}}{F_{w'}L_w} \quad (5-25a)
\]

\[
\text{where } F_{w'} = \phi_s F_s
\]

\[
F_s = 0.60F_{exx}
\]

\[
\phi_s = 0.75
\]

\[
\Omega = 2.00
\]

Additional terms used in the foregoing are defined as follows:

\[
\delta = 1 - \frac{d_s}{p}
\]

\[
\alpha' = 1.0 \text{ if } \beta \geq 1
\]

\[
= \text{lesser of } 1 \text{ and } \frac{1}{\delta} \left( \frac{2}{1-\beta} \right) \text{ if } \beta < 1
\]

For LRFD

\[
\beta = \left[ \frac{\phi_{n-1}}{P_n/n} - 1 \right] \frac{1}{\rho}
\]

For ASD

\[
\beta = \left[ \frac{r_n}{\Omega} - 1 \right] \frac{1}{\rho}
\]

\[
\rho = \frac{b'}{a'}
\]

$L_w = \text{HSS perimeter} = 2(B + H)$

\[
a' = a + \frac{d_s}{2}
\]

$a_n = \text{distance from bolt centerline to edge of the end plate but } \\
\leq 1.25b \text{ for calculation purposes}$

\[
a_n = a \leq 1.25b
\]

\[
b' = b - \frac{d_s}{2}
\]

$p = \text{length of end plate tributary to each bolt, in.}$

(See definition for the “no prying” case.)
5.7 CONNECTION DESIGN EXAMPLES

Example 5.1—End Tee Connection

Given:

Determine the applied load, \( P_r \), the connection can withstand in tension and compression for the end connection shown in Figure 5-9. Use four bolts, 1 in. diameter, ASTM A490-N, in standard holes and 70-ksi filler metal. The built-up tee stem is attached to an identical plate that is welded to the support structure.

From AISC Manual Tables 2-3 and 2-4, the material properties are as follows:

- **HSS8×6×5/16**
  - ASTM A500 Grade B
  - \( F_y = 46 \text{ ksi} \)
  - \( F_u = 58 \text{ ksi} \)

- **Plates**
  - ASTM A36
  - \( F_{yp} = 36 \text{ ksi} \)
  - \( F_{up} = 58 \text{ ksi} \)

From AISC Manual Table 1-11, the HSS geometric properties are as follows:

- **HSS8×6×5/16**
  - \( A = 7.59 \text{ in.}^2 \)
  - \( B = 6.00 \text{ in.} \)
  - \( H = 8.00 \text{ in.} \)
  - \( t = 0.291 \text{ in.} \)

- **Cap plate**
  - \( t_p = 1.00 \text{ in.} \)

- **Stem and support plates**
  - \( t_s = 0.750 \text{ in.} \)

![Diagram](image_url)

Fig. 5-9. Tee-to-HSS end connection.
Solution:

Available strength of the welds connecting the tee flange to the HSS

The nominal weld strength is determined as follows:

$$R_n = F_w A_w$$

(Spec. Eq. J2-3)

Because the force is not in the plane of the weld group, Equation J2-5 of the AISC Specification does not apply.

From AISC Specification Table J2.5, the nominal strength of the weld metal per unit area is:

$$F_w = 0.60 F_{txt}$$

$$= 0.60 \left( 70 \text{ ksi} \right)$$

$$= 42.0 \text{ ksi}$$

Assume the force from the stem spreads on both sides through the cap plate at a 2.5:1 slope. From AISC Specification Commentary Section K1.6, this produces a dispersed load width of $5t_p + N$, where $N = t_e + 2w$. When this load width equals or exceeds the HSS wall width, $B$, the four walls of the HSS are engaged. This is determined as follows:

$$\text{width} = 5t_p + t_e + 2w$$

$$= 5 \left( 1.00 \text{ in.} \right) + 0.750 \text{ in.} + 2 \left( \frac{7}{6} \text{ in.} \right)$$

$$= 6.38 \text{ in.} > B = 6.00 \text{ in.}$$

Therefore, all four HSS walls are fully effective so use an effective weld length of $l = 2(8.00 \text{ in.} + 6.00 \text{ in.}) = 28.0 \text{ in.}$

$$A_w = \frac{l w}{\sqrt{2}}$$

$$= 28.0 \text{ in.} \left( \frac{\frac{7}{6} \text{ in.}}{\sqrt{2}} \right)$$

$$= 6.19 \text{ in.}^2$$

$$R_n = 42.0 \text{ ksi} \left( 6.19 \text{ in.}^2 \right)$$

$$= 260 \text{ kips}$$

From AISC Specification Section J2.4, the available strength of the weld is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_n = 0.75 \left( 260 \text{ kips} \right)$</td>
<td>$R_n = 260 \text{ kips}$</td>
</tr>
<tr>
<td>= 195 kips</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td></td>
<td>$= 130 \text{ kips}$</td>
</tr>
</tbody>
</table>

Available strength of the welds connecting the stem to the tee flange

From AISC Specification Section J2.4 and Table J2.5, the nominal strength of the weld metal is determined as follows:

$$R_n = F_w A_w$$

(Spec. Eq. J2-4)

$$F_w = 0.60 F_{txt}$$

$$= 0.60 \left( 70 \text{ ksi} \right)$$

$$= 42.0 \text{ ksi}$$
Here the effective weld length, \( l \), is equal to the length of the flange plate, on both sides of the stem. Hence:

\[
\begin{align*}
  l &= 2(9.00 \text{ in.}) = 18.0 \text{ in.} \\
  A_w &= l \frac{w}{\sqrt{2}} \\
  &= 18.0 \text{ in.} \left(\frac{\frac{3}{16}\text{ in.}}{\sqrt{2}}\right) \\
  &= 3.98 \text{ in.}^2 \\
  R_n &= 42.0 \text{ ksi} \left(3.98 \text{ in.}^2\right) \\
  &= 167 \text{ kips}
\end{align*}
\]

From AISC Specification Section J2.4, the available strength of the weld is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.75(167 \text{ kips}) )</td>
<td>( R_n = 167 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>( \frac{R_n}{\Omega} = \frac{167 \text{ kips}}{2.00} = 83.5 \text{ kips} )</td>
</tr>
</tbody>
</table>

Local yielding of the HSS wall

From AISC Specification Section K1.6, determine the dispersed load width as follows:

\[
\begin{align*}
  N &= t_r + 2w \\
  &= 0.750 \text{ in.} + 2\left(\frac{3}{16} \text{ in.}\right) \\
  &= 1.38 \text{ in.} \\
  5t_r + N &= 5\left(1.00 \text{ in.}\right) + 1.38 \text{ in.} \\
  &= 6.38 \text{ in.} > 6.00 \text{ in.}
\end{align*}
\]

Therefore, the available strength is the sum of the contribution of all four walls. The nominal strength is:

\[
\begin{align*}
  R_n &= F_n A \\
  &= 46 \text{ ksi} \left(7.59 \text{ in.}^2\right) \\
  &= 349 \text{ kips}
\end{align*}
\]

From AISC Specification Section K1.6, the available strength of the HSS walls for the limit state of local yielding is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 1.00 )</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>( \phi R_n = 1.00(349 \text{ kips}) )</td>
<td>( R_n = 349 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>( \frac{R_n}{\Omega} = \frac{349 \text{ kips}}{1.50} = 233 \text{ kips} )</td>
</tr>
</tbody>
</table>
Shear yielding through the cap-plate thickness

From AISC Specification Section J4.2, the nominal shear yield strength is determined as follows:

\[ R_n = 0.60 F_y A_y \]  \hspace{1cm} \text{(Spec. Eq. J4-3)}

where

\[ F_y = F_{yp} \]
\[ = 36 \text{ ksi} \]

Note that the total shear area is twice the length of the cap plate times the thickness of the cap plate. Half of the load is resisted at each shear plane adjacent to the welds. The total shear area is:

\[ A_y = 2L_t \]
\[ = 2(9.00 \text{ in.})(1.00 \text{ in.}) \]
\[ = 18.0 \text{ in.}^2 \]

and therefore

\[ R_n = 0.60(36 \text{ ksi})(18.0 \text{ in.}^2) \]
\[ = 389 \text{ kips} \]

From AISC Specification Section J4.2, the available strength of the cap plate for the limit state of shear yielding is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi = 1.00 ]</td>
<td>[ \Omega = 1.50 ]</td>
</tr>
</tbody>
</table>
| \[ \phi R_n = 1.00(389 \text{ kips}) \] | \[ \frac{R_n}{\Omega} = \frac{389 \text{ kips}}{1.50} \]
| \[ = 389 \text{ kips} \] | \[ = 259 \text{ kips} \] |

Shear rupture through the cap-plate thickness

From AISC Specification Section J4.2, the nominal shear rupture strength is determined as follows:

\[ R_n = 0.60 F_u A_{uv} \]  \hspace{1cm} \text{(Spec. Eq. J4-4)}

where

\[ F_u = F_{up} \]
\[ = 58 \text{ ksi} \]
\[ A_{uv} = A_y \]
\[ = 18.0 \text{ in.}^2 \]

and therefore

\[ R_n = 0.60(58 \text{ ksi})(18.0 \text{ in.}^2) \]
\[ = 626 \text{ kips} \]

From AISC Specification Section J4.2, the available strength of the cap plate based on the limit state of shear rupture is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi = 0.75 ]</td>
<td>[ \Omega = 2.00 ]</td>
</tr>
</tbody>
</table>
| \[ \phi R_n = 0.75(626 \text{ kips}) \] | \[ \frac{R_n}{\Omega} = \frac{626 \text{ kips}}{2.00} \]
| \[ = 470 \text{ kips} \] | \[ = 313 \text{ kips} \] |
Bolt shear

From AISC Manual Table 7-1, the available shear strength per bolt is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi, r_v = 35.3$ kips/bolt</td>
<td>$r_v = 23.6$ kips/bolt</td>
</tr>
<tr>
<td>$\phi, R_v = 4(\phi, r_v)$</td>
<td>$R_v = 4 \left( \frac{r_v}{\Omega_v} \right)$</td>
</tr>
<tr>
<td>$\phi, R_v = 4(35.3$ kips/bolt)</td>
<td></td>
</tr>
<tr>
<td>= 141 kips</td>
<td>= 4(23.6 kips/bolt)</td>
</tr>
</tbody>
</table>

Bolt bearing

From AISC Specification Section J3.10(a)(i), the nominal bearing strength of a bolt in a standard hole when deformation at the bolt hole at service load is a design consideration is:

$$R_n = 1.2L_t F_u \leq 2.4dt F_u$$  \hspace{1cm} (Spec. Eq. J3-6a)

where

- $t = t_s = 0.750$ in.
- $F_u = F_w = 58$ ksi

For the end bolts

- $L_t = 3.00$ in. $- 1/16$ in.$/2 = 2.47$ in.

and therefore, the left side of the inequality in Equation J3-6a is:

$$1.2L_t F_u = 1.2(2.47 \text{ in.})(0.750 \text{ in.})(58 \text{ ksi}) = 129 \text{ kips}$$

The right side of the inequality in Equation J3-6a is:

$$2.4dt F_u = 2.4(1.00 \text{ in.})(0.750 \text{ in.})(58 \text{ ksi}) = 104 \text{ kips}$$

129 kips > 104 kips

Therefore, use $R_n = 104$ kips.
Using AISC Specification Section J3.10 for the end bolts and AISC Manual Table 7-5 for the interior bolts, the available bearing strength is determined as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| For the end bolts  
φ = 0.75  
φRn = 0.75(104 kips)  
= 78.0 kips  
For the interior bolts  
φr, = 101 kips per inch of thickness  
φRn = 101 kips/in.(0.750 in.)  
= 75.8 kips  
For the 4 bolts  
φRn = 2(78.0 kips) + 2(75.8 kips)  
= 308 kips  
| For the end bolts  
Ω = 2.00  
Rn = 104 kips  
= 52.0 kips  
For the interior bolts  
Ω = 67.4 kips per inch of thickness  
Rn = 67.4 kips/in.(0.750 in.)  
= 50.6 kips  
For the 4 bolts  
Rn = 2(52.0 kips) + 2(50.6 kips)  
= 205 kips |

**Tensile yielding of the tee stem**

From AISC Specification Section J4.1(a), the nominal tensile yield strength of the tee stem is:

\[ R_n = F_y A_y \]  

(Spec. Eq. J4-1)

where  
\[ F_y = F_{yw} \]  
= 36 ksi  
\[ A_y = Wt_s \]  
= 9.00 in.(0.750 in.)  
= 6.75 in.²

and therefore  
\[ R_n = 36 \text{ ksi}(6.75 \text{ in.}²) \]  
= 243 kips

From AISC Specification Section J4.1(a), the available tensile yield strength of the tee stem is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| φ = 0.90  
φRn = 0.90(243 kips)  
= 219 kips  
| Ω = 1.67  
Rn = 243 kips  
= 146 kips |
Tensile rupture of the tee stem

From AISC Specification Section J4.1(b), the nominal tensile rupture strength of the tee stem is:

\[ R_n = F_u A_e \]  

(Spec. Eq. J4-2)

where \[ F_u = F_{upp} \]

= 58 ksi

\[ A_e = A_e U \]  

(Spec. Eq. D3-1)

\[ U = 1.0 \] from AISC Specification Table D3.1 (no unconnected elements)

From AISC Specification Section J4.1(b), the net tensile area is:

\[ A_n = A_e - 2 \left( d_a + \frac{1}{16} \text{ in.} \right) t_s \leq 0.85 A_y \]

where

\[ d_a + \frac{1}{16} \text{ in.} = 1.00 \text{ in.} + \frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.} \]

= 1 1/16 in.

The net tensile area is:

\[ A_n = 6.75 \text{ in.}^2 - 2 \left( 1.13 \text{ in.} \right) \left( 0.750 \text{ in.} \right) \]

= 5.06 in.\(^2\) \leq 0.85 A_y

\[ \leq 0.85 \left( 6.75 \text{ in.}^2 \right) = 5.74 \text{ in.}^2 \]

and the effective net area is:

\[ A_e = 5.06 \text{ in.}^2 \left( 1.0 \right) \]

= 5.06 \text{ in.}^2

The nominal tensile rupture strength of the tee stem is:

\[ R_n = 58 \text{ ksi} \left( 5.06 \text{ in.}^2 \right) \]

= 293 kips

From AISC Specification Section J4.1(b), the available tensile rupture strength of the tee stem is:

\[ \frac{R_n}{\Omega} = 293 \text{ kips} \]

\[ \Omega = 2.00 \]

\[ = 147 \text{ kips} \]

Block shear rupture of the tee stem plate

From AISC Specification Section J4.3, the nominal strength of the tee stem based on the limit state of block shear rupture is:

\[ R_n = 0.6F_y A_{yw} + U_{uy} F_y A_{uy} \leq 0.6F_y A_{yw} + U_{uy} F_y A_{uy} \]  

(Spec. Eq. J4-5)
where
\[ A_{gv} = 2L_{cov}t_i \]
\[ L_{cv} = 3.00 \text{ in.} + 3.00 \text{ in.} = 6.00 \text{ in.} \]
\[ A_{gv} = 2(6.00 \text{ in.})(0.750 \text{ in.}) = 9.00 \text{ in}^2 \]
\[ A_{mv} = A_{gv} - 2(1.5)(d_h + y_i) \]
\[ = 9.00 \text{ in}^2 - 2(1.5)(1/16 \text{ in.} + 1/16 \text{ in.})(0.750 \text{ in.}) = 6.47 \text{ in}^2 \]
\[ A_{mv} = t_i[3.00 - (d_h + y_i)] \]
\[ = 0.750 \text{ in.} - (1/16 \text{ in.} + 1/16 \text{ in.}) = 1.41 \text{ in.}^2 \]
\[ U_{lv} = 1.0 \text{ since tension is uniform} \]

The left side of the inequality given in AISC Specification Equation J4-5 is:
\[ 0.6F_A A_{mv} + U_{lv} F_A A_{mv} = 0.6(58 \text{ ksi})(6.47 \text{ in}^2) + 1.0(58 \text{ ksi})(1.41 \text{ in}^2) = 307 \text{kips} \]

The right side of the inequality given in Equation J4-5 is:
\[ 0.6F_A A_{mv} + U_{lv} F_A A_{mv} = 0.6(36 \text{ ksi})(9.00 \text{ in}^2) + 1.0(58 \text{ ksi})(1.41 \text{ in}^2) = 276 \text{kips} \]

Because 307 kips > 276 kips, use \( R_u = 276 \text{kips} \).

The available strength of the tee stem for the limit state of block shear rupture is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_u = 0.75(276 \text{kips}) )</td>
<td>( \frac{R_u}{\Omega} = \frac{276 \text{kips}}{2.00} = 138 \text{kips} )</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The available strength in tension is therefore controlled by the available strength of the weld connecting the stem to the tee flange.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = 125 \text{kips} )</td>
<td>( \frac{R_u}{\Omega} = 83.5 \text{kips} )</td>
</tr>
</tbody>
</table>

Additional limit states for the available compressive strength are:

**HSS wall local crippling**

From AISC Specification Section K1.6(ii), the nominal compressive strength based on the limit state of wall local crippling, for one wall, is:

\[ R_u = 0.80t^2 \left[1 + \frac{6N}{B} \left( \frac{t}{t_p} \right)^{15} \right] \left( \frac{EF_{tp}}{t} \right) \]  
*(Spec. Eq. K1-12)*
where
\[ N = t_s + 2w \]
\[ = 0.750 \text{ in.} + 2\left(\frac{\frac{3}{8}}{2}\right) \text{ in.} \]
\[ = 1.38 \text{ in.} \]

The nominal compressive strength due to wall local crippling is:
\[ R_n = 0.80\left(0.291 \text{ in.}\right)^2 \left[ 1 + 6\left(1.38 \text{ in.}\right)\left(0.291 \text{ in.}\right)\left(\frac{6.00 \text{ in.}}{1.00 \text{ in.}}\right)\right]^{13} \sqrt{29,000 \text{ ksi}}\left(46 \text{ ksi}\right)\left(1.00 \text{ in.}\right) \]
\[ = 176 \text{ kips per 6.00 in. HSS wall} \]
\[ 2R_n = 2(176 \text{ kips}) \]
\[ = 352 \text{ kips for two 6.00 in. HSS walls} \]

From AISC Specification Section K1.6(ii), the available compressive strength of the HSS for the limit state of wall local crippling is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.75(352 \text{ kips}) )</td>
<td>( \frac{R_n}{\Omega} = \frac{352 \text{ kips}}{2.00} )</td>
</tr>
<tr>
<td>( = 264 \text{ kips} )</td>
<td>( = 176 \text{ kips} )</td>
</tr>
</tbody>
</table>

Buckling of the stem and interaction for combined flexure and compression

The effect of combined flexure and compression can be determined using Equations 5-3 and 5-4. Assuming that \( P_c/P_r \geq 0.2 \), use the following to determine the maximum required compressive strength, \( P_r \):

\[ P_r = \frac{1}{\frac{4e}{P_c} + \frac{9M_c}{9M_c}} \]  \hspace{1cm} (5-3)

where
\[ e = \frac{0.750 \text{ in.} + 0.750 \text{ in.}}{2} \]
\[ = 0.750 \text{ in.} \]

Determine the available compressive strength, \( P_c \). The slenderness ratio, \( KL/r \), is first calculated as follows:
\[ r = \frac{t_s}{\sqrt{12}} \]
\[ = \frac{0.750 \text{ in.}}{\sqrt{12}} \]
\[ = 0.217 \text{ in.} \]

Therefore
\[ KL/r = \frac{1.2\left(3.00 \text{ in.} + 3.00 \text{ in.} + 3.00 \text{ in.}\right)}{0.217 \text{ in.}} \]
\[ = 49.8 \]
Enter AISC Manual Table 4-22 with this value for $KL/r$ and determine the available critical stress, which can then be used to determine the available compressive strength, $P_c$, as follows:

\[
\begin{array}{c|c}
\text{LRFD} & \text{ASD} \\
\hline
\phi, F_{cr} = 28.4 \text{ ksi} & F_{cr} = 18.9 \text{ ksi} \\
P_c = \frac{F_{cr}}{\Omega_e} & P_c = \frac{P_a}{\Omega_e} \\
= \phi, F_{cr} A_g & = \frac{F_{cr}}{\Omega_e} (A_g) \\
\end{array}
\]

where
\[A_g = L t_s\]
\[= 9.00 \text{ in.} \left(0.750 \text{ in.}\right)^2\]
\[= 6.75 \text{ in.}^2\]

\[
\begin{array}{c|c}
\text{LRFD} & \text{ASD} \\
\hline
P_c = 28.4 \text{ ksi} \left(6.75 \text{ in.}^2\right) & P_c = 18.9 \text{ ksi} \left(6.75 \text{ in.}^2\right) \\
= 192 \text{ kips} & = 128 \text{ kips} \\
\end{array}
\]

Using AISC Specification Section F7.1, the available flexural strength, $M_c$, for a compact HSS is:

\[
\begin{array}{c|c}
\text{LRFD} & \text{ASD} \\
\hline
\phi_c = 0.90 & \Omega_s = 1.67 \\
M_c = \phi_c F_{cr} Z & M_c = \frac{F_{cr} Z}{\Omega_s} \\
\end{array}
\]

where
\[Z = \frac{L t_s^2}{4}\]
\[= 9.00 \text{ in.} \left(0.750 \text{ in.}\right)^2 \\
= 1.27 \text{ in.}^3\]

\[
\begin{array}{c|c}
\text{LRFD} & \text{ASD} \\
\hline
M_c = 0.90 \left(36 \text{ ksi} \right) \left(1.27 \text{ in.}^3\right) & M_c = \frac{36 \text{ ksi} \left(1.27 \text{ in.}^3\right)}{1.67} \\
= 41.1 \text{ kip-in.} & = 27.4 \text{ kip-in.} \\
\end{array}
\]
Therefore, the required compressive strength, $P_r$, is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_r = \frac{1}{P_e + \frac{4e}{9M_e}}$</td>
<td>$P_e = \frac{1}{P_e + \frac{4e}{9M_e}}$</td>
</tr>
<tr>
<td>(5-3)</td>
<td>(5-3)</td>
</tr>
<tr>
<td>$= \frac{1}{192 \text{ kips} + \frac{4(0.750 \text{ in.})}{9(41.1 \text{ kip-in.})}}$</td>
<td>$= \frac{1}{128 \text{ kips} + \frac{4(0.750 \text{ in.})}{9(27.4 \text{ kip-in.})}}$</td>
</tr>
<tr>
<td>$= 75.1 \text{ kips}$</td>
<td>$= 50.1 \text{ kips}$</td>
</tr>
<tr>
<td>$\frac{P_r}{P_e} = 75.1 \text{ kips}$</td>
<td>$\frac{P_e}{P_e} = 128 \text{ kips}$</td>
</tr>
<tr>
<td>$= 0.391 &gt; 0.2 \text{ o.k.}$</td>
<td>$= 0.391 &gt; 0.2 \text{ o.k.}$</td>
</tr>
</tbody>
</table>

The available compressive strength is therefore controlled by buckling of the stem.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_a = 75.1 \text{ kips}$</td>
<td>$\frac{R_a}{\Omega} = 50.1 \text{ kips}$</td>
</tr>
</tbody>
</table>

**Example 5.2—Slotted Round HSS/Gusset Connection**

**Given:**

Determine the available tensile strength for the HSS in the connection illustrated in Figure 5-10. The slot in the HSS for the gusset plate is concentric and is $\frac{1}{6}$ in. wider than the plate thickness to allow for clearance. Laboratory tests on such statically loaded connections (Martinez-Saucedo and Packer, 2006) have shown that the end of the slot in the HSS need not be machined or drilled to a smooth radius. The four bolts are 3/8-in.-diameter ASTM A325-N in standard holes.

---

**Fig. 5-10. Slotted round HSS to concentric gusset plate connection.**
From AISC *Manual* Tables 2-3 and 2-4, the material properties are as follows:

**HSS 6.000 x 0.250**

- ASTM A500 Grade B
- $F_y = 42$ ksi
- $F_u = 58$ ksi

**Plate**

- ASTM A36
- $F_{yp} = 36$ ksi
- $F_{up} = 58$ ksi

From AISC *Manual* Table 1-13, the HSS geometric properties are as follows:

<table>
<thead>
<tr>
<th>HSS 6.000 x 0.250</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A = 4.22$ in.²</td>
</tr>
<tr>
<td>$D = 6.00$ in.</td>
</tr>
<tr>
<td>$t = 0.233$ in.</td>
</tr>
</tbody>
</table>

The gusset plate geometric properties are as follows:

- Gusset plate
- $W = 7.00$ in.
- $t_p = 0.375$ in.

**Solution:**

**HSS tensile yielding**

From AISC *Specification* Section D2, the nominal tensile strength for tensile yielding in the gross section is:

$$P_n = F_y A_g$$  \hspace{1cm} (*Spec. Eq. D2-1*)

where

$$A_g = A = 4.22 \text{ in.}^2$$

The nominal tensile strength is:

$$P_n = 42 \text{ ksi} \left(4.22 \text{ in.}^2\right) = 177 \text{ kips}$$  \hspace{1cm} (*Spec. Eq. D2-1*)

The available tensile strength for tensile yielding in the gross section is:

$$P_n = 177 \text{ kips}$$  \hspace{1cm} (*ASD*)

**HSS tensile rupture**

From AISC *Specification* Section D2, the nominal tensile strength for tensile rupture in the net section of the HSS is:

$$P_n = F_u A_n$$  \hspace{1cm} (*Spec. Eq. D2-2*)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.90$</td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$\phi P_n = 0.90(177 \text{ kips})$</td>
<td>$P_n \Omega = 177 \text{ kips}$</td>
</tr>
<tr>
<td>$= 159 \text{ kips}$</td>
<td>$= 106 \text{ kips}$</td>
</tr>
</tbody>
</table>
where the effective area, $A_e$, of tension members is determined from AISC Specification Section D3.3 as follows:

$$A_e = A_e U$$

$$A_e = A - 2 \left( t_p + \frac{1}{32} \text{ in.} \right) t$$ for a slot clearance of $\frac{1}{32} \text{ in.}$

$$= 4.22 \text{ in.}^2 - 2 \left( 0.375 \text{ in.} + \frac{1}{32} \text{ in.} \right) \left( 0.233 \text{ in.} \right)$$

$$= 4.02 \text{ in.}^2$$

When $D \leq l < 1.3D$, $U = 1 - \frac{\bar{x}}{l}$ from AISC Specification Table D3.1, Case 5.

$$l = D$$

$$= 6.00 \text{ in.}$$

$$\bar{x} = \frac{D}{\pi}$$

$$= \frac{6.00}{\pi}$$

$$= 1.91 \text{ in.}$$

$$U = 1 - \frac{\bar{x}}{l}$$

$$= 1 - \frac{1.91 \text{ in.}}{6.00 \text{ in.}}$$

$$= 0.682$$

$$A_e = 4.02 \text{ in.}^2 \left( 0.682 \right)$$

$$= 2.74 \text{ in.}^2$$

Therefore, the available tensile strength for the limit state of tensile rupture in the net section is determined as follows:

$$P_n = 58 \text{ ksi} \left( 2.74 \text{ in.}^2 \right)$$

$$= 159 \text{ kips}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi P_n = 0.75 \left( 159 \text{ kips} \right)$</td>
<td>$\frac{P_n}{\Omega} = \frac{159 \text{ kips}}{2.00}$</td>
</tr>
<tr>
<td>$= 119 \text{ kips}$</td>
<td>$= 79.5 \text{ kips}$</td>
</tr>
</tbody>
</table>

**Gusset plate tensile yielding**

From AISC Specification Section J4, the nominal tensile strength for the limit state of tensile yielding of the gusset plate is:

$$R_n = F_y A_t$$

$$(Spec. \text{ Eq. J4-1})$$

where

$$F_y = F_{wp}$$

$$= 36 \text{ ksi}$$

$$A_t = W_t$$

$$= 7.00 \text{ in.} \left( 0.375 \text{ in.} \right)$$

$$= 2.63 \text{ in.}^2$$
Therefore, the nominal tensile strength is:

\[ R_n = 36 \text{ ksi} \left( 2.63 \text{ in.}^2 \right) \]

\[ = 94.7 \text{ kips} \]

The available tensile strength for the limit state of tensile yielding of the gusset plate is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.90(94.7 \text{ kips}) )</td>
<td>( R_n = 94.7 \text{ kips} )</td>
</tr>
<tr>
<td>[ = 85.2 \text{ kips} ]</td>
<td>[ \Omega = 1.67 ]</td>
</tr>
</tbody>
</table>

**Gusset plate tensile rupture**

From AISC Specification Section J4, the nominal tensile strength for the limit state of tensile rupture of the gusset plate is:

\[ R_n = F_y A_e \]

where

\( F_y = 58 \text{ ksi} \)

The effective area, \( A_e \), is determined from AISC Specification Section D3.3 as follows:

\[ A_e = A_U \]

The net area, \( A_n \), is determined from AISC Specification Section D3.2 as follows:

\[ A_n = A_e - 2(d_z + \frac{1}{6} \text{ in.})t_p \]

where

\( d_z = \frac{3}{8} \text{ in.} + \frac{1}{6} \text{ in.} \)

\[ = 0.938 \text{ in.} \]

Therefore, the net area is:

\[ A_n = 2.63 \text{ in.}^2 - 2 \left( 0.938 \text{ in.} + 0.063 \text{ in.} \right) \left( 0.375 \text{ in.} \right) \]

\[ = 1.88 \text{ in.}^2 \]

From AISC Specification Table D3.1, with no unconnected elements, \( U = 1.0 \).

According to AISC Specification Section J4.1(b), the effective net area is limited to \( 0.85A_n \) for bolted splice plates. Treating the gusset plate as a splice plate gives:

\[ A_e = 1.88 \text{ in.}^2 \left( 1.00 \right) \]

\[ = 1.88 \text{ in.}^2 \leq 0.85A_n \]

\[ 0.85 \left( 2.63 \text{ in.}^2 \right) = 2.24 \text{ in.}^2 \]

\[ 1.88 \text{ in.}^2 \leq 2.24 \text{ in.}^2 \]

The nominal tensile strength for the limit state of tensile rupture of the gusset plate is:

\[ R_n = 58 \text{ ksi} \left( 1.88 \text{ in.}^2 \right) \]

\[ = 109 \text{ kips} \]
The available tensile strength for the limit state of tensile rupture of the gusset plate is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.75(109 \text{ kips}) )</td>
<td>( \frac{R_n}{\Omega} = \frac{109 \text{ kips}}{2.00} = 54.5 \text{ kips} )</td>
</tr>
<tr>
<td>( = 81.8 \text{ kips} )</td>
<td>( \Omega = 2.00 )</td>
</tr>
</tbody>
</table>

### Block shear rupture of the gusset plate

From AISC Specification Section J4.3, the nominal strength for the limit state of block shear rupture of the gusset plate is:

\[
R_n = 0.6F_u A_w + U_u F_s A_{sv} \leq 0.6F_s A_{sv} + U_u F_s A_{sw} \tag{Spec. Eq. J4-5}
\]

where

- \( F_u = F_{up} \)
- \( F_s = F_{sp} \)
- \( U_u = 1 \) (uniform stress)
- \( A_{sw} = 2 \left( 1.50 \text{ in.} + 3.00 \text{ in.} \right) \left( 0.375 \text{ in.} \right) = 3.38 \text{ in.}^2 \)
- \( A_{sv} = A_{sv} - 2 \left[ \text{ deduction for (1 + } \frac{1}{2} \text{) bolt holes} \right] F_s \)
  - \( = 3.38 \text{ in.}^2 - 2 \left( 1.5 \left( \frac{7}{8} \text{ in.} + \frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.} \right) \left( 0.375 \text{ in.} \right) = 2.26 \text{ in.}^2 \)
- \( A_{sv} = \left[ 3.00 \text{ in.} - 2 \left( 0.5 \left( \frac{7}{8} \text{ in.} + \frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.} \right) \right) \left( 0.375 \text{ in.} \right) \right]
  - \( = 0.750 \text{ in.}^2 \)

The left side of the inequality given in AISC Specification Equation J4-5 is:

\[
0.6F_u A_w + U_u F_s A_{sv} = 0.6 \left( 58 \text{ ksi} \right) \left( 2.26 \text{ in.}^2 \right) + 1.00 \left( 58 \text{ ksi} \right) \left( 0.750 \text{ in.}^2 \right) = 122 \text{ kips}
\]

The right side of the inequality given in AISC Specification Equation J4-5 is:

\[
0.6F_s A_{sv} + U_u F_s A_{sw} = 0.6 \left( 36 \text{ ksi} \right) \left( 3.38 \text{ in.}^2 \right) + 1.00 \left( 58 \text{ ksi} \right) \left( 0.750 \text{ in.}^2 \right) = 117 \text{ kips}
\]

Because \( 122 > 117 \) kips, use \( R_n = 117 \) kips. The available strength of the gusset plate for the limit state of block shear rupture is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.75(117 \text{ kips}) )</td>
<td>( \frac{R_n}{\Omega} = \frac{117 \text{ kips}}{2.00} = 58.5 \text{ kips} )</td>
</tr>
<tr>
<td>( = 87.8 \text{ kips} )</td>
<td>( \Omega = 2.00 )</td>
</tr>
</tbody>
</table>
Shear of the weld metal and shear of the HSS and gusset plate base metals

Because the HSS slot is \( \frac{5}{16} \) in. wider than the gusset thickness, the gap for either weld will not exceed \( \frac{5}{16} \) in., and no adjustment of the weld size is required. Therefore, the effective weld size is the full weld: \( w_{\text{eff}} = w = \frac{5}{16} \) in. From Table 2-3, \( t_{\text{min}} = 0.166 \) in. for ASTM A500 Grade B HSS with a \( \frac{5}{16} \) in. weld size. The design thickness of the HSS is \( t = 0.233 \) in., which is greater than \( t_{\text{min}} = 0.166 \) in. Therefore, the shear strength of the weld metal controls over the shear strength of the HSS base metal.

The gusset plate has two shear planes. Because \( F_{\text{yy}} / F_{\text{up}} = 0.621 < 0.750 \), the limit state of shear yielding controls over the limit state of shear rupture. As derived in Section 2.3, the effective weld size is:

\[
D_{\text{eff}} = 30.2 \left( \frac{F_{\text{yy}}}{F_{\text{yy}}} \right) t_{\text{min}}
\]

\[
= 3.00 \text{ sixteenths-of-an-inch}
\]

Solve for \( t_{\text{min}} \):

\[
t_{\text{min}} = \frac{D_{\text{eff}}}{30.2} \left( \frac{F_{\text{yy}}}{F_{\text{yy}}} \right)
\]

\[
= 3.00 \left( \frac{70 \text{ ksi}}{36 \text{ ksi}} \right)
\]

\[
= 0.193 \text{ in.}
\]

\( t_p = 0.375 \text{ in.} < 2t_{\text{min}} \)

\( 2t_{\text{min}} = 2(0.193 \text{ in.}) = 0.386 \text{ in.} \)

\( 0.375 < 0.386 \)

Therefore, shear yielding of the gusset plate base metal controls over the weld metal. From AISC Specification Section J2.4, the nominal strength of the base metal for the limit state of shear yielding is:

\[
R_n = F_{\text{BM}} A_{\text{BM}}
\]  \hspace{1cm} (Spec. Eq. J2-2)

AISC Specification Table J2.5 stipulates that the available shear strength of the base metal is governed by Section J4. From Section J4.2(a), the nominal shear strength for the limit state of shear yielding of the gusset plate is:

\[
R_n = 0.60 F_{\text{yy}} A_g
\]  \hspace{1cm} (Spec. Eq. J4-3)

Therefore, the variables in Equation J2-2 are defined as follows:

\[
F_{\text{BM}} = 0.6 F_{\text{yy}}
\]

\[
= 0.6(36 \text{ ksi}) = 21.6 \text{ ksi}
\]

The cross-sectional area of the base metal at the weld is:

\[
A_{\text{BM}} = 2t_p
\]

\[
= 2(6.00 \text{ in.})(0.375 \text{ in.})
\]

\[
= 4.50 \text{ in.}^2
\]

Therefore, the nominal strength of the base metal for the limit state of shear yielding is:

\[
R_n = 21.6 \text{ ksi}(4.50 \text{ in.}^2) = 97.2 \text{ kips}
\]
The available strength of the base metal for the limit state of shear yielding is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 1.00$</td>
<td>$\Omega = 1.50$</td>
</tr>
<tr>
<td>$\phi R_n = 1.00 \times (97.2 \text{ kips})$</td>
<td>$R_n = \frac{97.2 \text{ kips}}{1.50}$</td>
</tr>
<tr>
<td>= 97.2 kips</td>
<td>= 64.8 kips</td>
</tr>
</tbody>
</table>

**Bolt shear**

From AISC Manual Table 7-1, the available shear strength for four $\frac{3}{8}$-in.-diameter ASTM A325-N bolts is determined as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi, r_n = 21.6 \text{ kips/bolt}$</td>
<td>$r_n = 14.4 \text{ kips/bolt}$</td>
</tr>
<tr>
<td>$\phi, R_n = 4 \left( \phi, r_n \right)$</td>
<td>$R_n = 4 \left( \frac{r_n}{\Omega_v} \right)$</td>
</tr>
<tr>
<td>$\phi, R_n = 4 \left( 21.6 \text{ kips/bolt} \right)$</td>
<td>= 4 \left( 14.4 \text{ kips/bolt} \right)</td>
</tr>
<tr>
<td>= 86.4 kips</td>
<td>= 57.6 kips</td>
</tr>
</tbody>
</table>

**Bolt bearing**

Assume the gusset plate is thinner than the plate member it is bolted to and will therefore control the strength. From AISC Specification Section J3.10(a)(i):

$$R_n = 1.2 L_c t F_v \leq 2.4 d t F_u \quad \text{(Spec. Eq. J3-6a)}$$

where

- $t = t_p$
  - = 0.375 in.
- $F_u = F_{wp}$
  - = 58 ksi

For the end bolts:

- $L_c = \text{clear distance, in the direction of the force, between the edge of the hole and the edge of the material, in.}$
  - = $1.50 - d_n / 2$
  - = 1.50 in. $- \frac{3}{16} \text{ in.} / 2$
  - = 1.03 in.

Therefore, the nominal bearing strength at the end bolt holes is:

$$R_n = 1.2 \left( 1.03 \text{ in.} \right) \left( 0.375 \text{ in.} \right) \left( 58 \text{ ksi} \right)$$

$$= 26.9 \text{ kips} < 2.4 \left( \frac{3}{8} \text{ in.} \right) \left( 0.375 \text{ in.} \right) \left( 58 \text{ ksi} \right)$$

$$= 45.7 \text{ kips}$$

Use $R_n = 26.9$ kips.
The available bearing strength for the four bolts is determined as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| For the end bolts  
$$\phi = 0.75$$  
$$\phi R_n = 0.75 \times 26.9 \text{ kips}$$  
$$= 20.2 \text{ kips}$$  
From AISC Manual Table 7-5, the available bearing strength for the interior bolts is:  
$$\phi R_n = 91.4 \text{ kips/in.} \times 0.375 \text{ in.}$$  
$$= 34.3 \text{ kips}$$  
The available bearing strength for the four bolts is:  
$$\phi R_n = 2 \times 20.2 \text{ kips} + 2 \times 34.3 \text{ kips}$$  
$$= 109 \text{ kips}$$  

For the end bolts  
$$\Omega = 2.00$$  
$$R_n = \frac{26.9 \text{ kips}}{2.00}$$  
$$= 13.5 \text{ kips}$$  
From AISC Manual Table 7-5, the available bearing strength for the interior bolts is:  
$$R_n = 60.9 \text{ kips/in.} \times 0.375 \text{ in.}$$  
$$= 22.8 \text{ kips}$$  
The available bearing strength for the four bolts is:  
$$R_n = 2 \times 13.5 \text{ kips} + 2 \times 22.8 \text{ kips}$$  
$$= 72.6 \text{ kips}$$

The available strength in tension is controlled by the limit state of gusset plate tensile rupture.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$$\phi R_n = 81.8 \text{ kips}$$</td>
<td>$$\frac{R_n}{\Omega} = 54.5 \text{ kips}$$</td>
</tr>
</tbody>
</table>
Chapter 6
Branch Loads on HSS—An Introduction

This brief chapter is intended to serve as an introduction to Chapters 7, 8 and 9, in which connection nominal strengths are tabulated, along with applicable limits of validity for the formulas. These design procedures are in accordance with Chapter K of the AISC Specification (AISC, 2005a) with some minor modifications. Classic failure modes for HSS welded connections, wherein the main (or through) HSS member is loaded by attached HSS branches or plates, are described in this chapter. This provides a physical understanding of the limit states that are to be checked in the following chapters and, importantly, allows the user of this Design Guide to understand HSS connection behavior and extrapolate “engineering judgment” to other connection types that are beyond the scope of this Design Guide. The chapter also provides some generic design guidance for HSS connections. Information on the connection classification for HSS-to-HSS truss connections can be found in Section 8.3, which discusses K-, N-, Y-, T- and X-connections.

6.1 PRINCIPAL LIMIT STATES

6.1.1 Chord or Column Wall Plastification

This failure mode is particularly prevalent in HSS connections due to the flexible nature of the connecting HSS face, which readily distorts under loading normal to the chord connecting surface. If the main member is a square or rectangular HSS, then the chord connecting face acts like a flat plate under transverse load, supported by two remote webs. The HSS connecting face acts relatively independent of the other three sides. Plastification of the connecting HSS face is the most common failure mode for gapped K- and N-connections with small to medium ratios of branch (or web) member widths to chord width (β). In the case of gapped K-connections, the actions of the compression branch and tension branch develop a “push–pull” mechanism on the chord connecting face, usually resulting in large deformations of the connecting face as shown in Figure 6-1. This limit state is the one specifically covered by AISC Specification Equation K2-20.

If the main member is a round HSS, then the chord behaves like a closed ring under transverse load and chord plastification results in distortion of the entire chord cross-section. The failure mode in Figure 6-2(a) is the one specifically covered by AISC Specification Equations K2-6 and K2-8. Chord plastification applies to many other HSS connection types, including T-, Y- and cross-connections under branch axial loading (AISC Specification Equations K2-3, K2-5 and K2-13); T-, Y- and cross-connections under branch moment loading (AISC Specification Equations K3-3, K3-5, K3-11 and K3-15); and branch plate-to-HSS connections (AISC Specification Equations K1-1, K1-8 and K1-9). Longitudinal plate-to-HSS connections under branch axial load are particularly susceptible to chord plastification, and Figure 6-2(b) shows the gross deformations that are achieved at the ultimate capacity of such a connection. As a result, the connection nominal strength for this limit state also includes a connection deformation control.

6.1.2 Chord Shear Yielding (Punching Shear)

This failure mode, often termed simply “punching shear,” may govern in various HSS connections, particularly for medium to high branch-to-chord width ratios. With this failure mechanism, a patch of chord material pulls out (or punches in) around the footprint of a branch member, at the toe of
the weld to the chord. Such a failure can occur under either a tension branch or a compression branch member, providing the branch is physically capable of shearing through the chord wall [i.e., the branch outside width (or diameter) is less than the inside width (or diameter) of the chord]. Shear failure strength is typically calculated in steel design codes based on either the ultimate shear stress \( (F_u/\sqrt{3} = 0.6F_u) \), with a relatively low resistance factor on the order of 0.75, or the shear yield stress \( (F_y/\sqrt{3} = 0.6F_y) \), with a resistance factor close to unity. The AISC Specification uses both of these approaches in Section J4 for “block shear,” checking the limit states of shear yielding and shear rupture. In Chapter K, however, the AISC Specification checks simply for shear yielding with a shear yield stress of \( 0.6F_y \) and a resistance factor of 0.95. The chord shear yielding limit state check applies to a considerable number of HSS connections: T-, Y- and cross-connections under branch axial loading (AISC Specification Equations K2-4 and K2-14); gapped K-connections (AISC Specification Equations K2-9 and K2-21); T-, Y- and cross-connections under branch moment loading (AISC Specification Equations K3-4 and K3-6); and transverse branch plate-to-HSS connections (AISC Specification Equation K1-3). One should note that the shear yield stress of \( 0.6F_y \) is not always applied to the entire perimeter around the footprint of a branch member—for transverse plates or transverse walls of a rectangular HSS across a rectangular HSS chord member, just an effective width is used (see AISC Specification Equations K1-3, K2-14 and K2-21). This is because the transverse plate (or transverse wall of an HSS branch) is not uniformly loaded across its width; it is very highly stressed at the outer portions of its width adjacent to the (stiff) sidewalls of the rectangular HSS chord member. As a consequence, the transverse plate (or transverse wall of an HSS branch) punches out the chord connecting face prematurely in these highly loaded regions [see Figure 6-3(a)]. For round HSS connections under branch axial loading, on the other hand, uniform punching shear is assumed around the footprint of the branch, commensurate with the failure mode shown in Figure 6-3(b).

### 6.1.3 Local Yielding Due to Uneven Load Distribution

This failure mode applies to transverse plates, or transverse walls of a rectangular HSS, across a rectangular HSS chord member. It is analogous to the plate effective punching shear width concept described for the previous failure mode, except now the effective width is applied to the transverse element itself rather than the HSS chord, resulting in premature failure of that element. In tension, local yielding and then premature failure of the transverse element occurs, as shown

(a) Balanced gapped K-connection at ultimate load.

(b) Longitudinal plate connection at ultimate load.

Fig. 6-2. Chord plastification for round HSS connections.

(a) Transverse plate-to-rectangular HSS connection at ultimate load. Note that rupture initiates at the plate extremities.

(b) Round HSS T-connection at ultimate load.

Fig. 6-3. Chord shear yielding (punching) for HSS connections.
in Figure 6-4(a). In compression, local yielding usually results in premature local buckling of the element. This is the most common failure mode for rectangular HSS overlapped K-connections, wherein local buckling of the compression branch occurs [see Figure 6-4(b)]. The local yielding due to uneven load distribution limit state check is applied to many HSS connections with rectangular chord members in the AISC Specification: T-, Y- and cross-connections under branch axial loading (AISC Specification Equation K2-18); gapped and overlapped K-connections (AISC Specification Equations K2-22, K2-24, K2-25 and K2-26); T- and cross-connections under branch moment loading (AISC Specification Equations K3-13 and K3-17); and transverse branch plate-to-HSS connections (AISC Specification Equation K1-2). An inspection of the latter equation (K1-2) shows that the effective width of a transverse element depends highly on the slenderness of the main HSS chord member—if the chord connecting face is thin and flexible (high B/t), then the effective width will be low. Conversely, maximum transverse element effective width is achieved for stocky chords (low B/t), but with an upper limit of the actual element width. This underlines the design principle (see Section 6.2) of achieving high HSS connection strengths by using stocky (thick-walled) chord members.

6.1.4 Chord or Column Sidewall Failure

Failure of the chord member sidewalls, rather than the connecting face, may occur in rectangular HSS connections when the branch width is close to, or equals, the chord member width (i.e., $\beta = 1.0$). Such connections are also often termed “matched box connections” [e.g., by AWS D1.1 Chapter 2 (AWS, 2006)]. If the branch is in tension, the failure mode of the chord is sidewall local yielding (AISC Specification Equations K1-4, K2-15, K3-12 and K3-16) over a dispersed load width. The same failure mechanism is also possible if the branch is in compression—particularly if the chord side walls are relatively stocky (low $H/t$) or if the bearing length is low [see Figure 6-5(a)]. Under branch compression loading, sidewall local crippling or sidewall local buckling are
also potential failure modes (AISC Specification Equations K1-5, K1-6, K2-16 and K2-17). The equations representing both of these limit states have been adapted (for two webs in compression) from concentrated loading to a W-section beam flange, elsewhere in the AISC Specification. Sidewall local buckling in a rectangular HSS cross-connection is shown in Figure 6-5(b), where the dark lines in the connection indicate regions of large plastic strain causing flaking of the whitewash. Figure 6-5(c) shows the overall web buckling mechanism, which is assumed to occur over a buckling length of \( H - 3t \) with welded branches. [Note that 1.5\( t \) is the minimum outside corner radius that hence produces the maximum flat length for the HSS wall, according to AISC Specification Section B4.2(d).]

Some other connection limit states are analyzed in Chapter 7 (for special connection types), Chapter 8 (shear failure of the chord member side walls for rectangular HSS), and Chapter 9 (chord distortional failure for rectangular HSS connections under out-of-plane bending) but these are much less common. One known failure mode for rectangular HSS K-connections, and in particular overlapped K-connections, is local buckling of the chord connecting face behind the heel of the tension branch, as illustrated in Figure 6-6. Such failure is caused by a shear lag effect, because a disproportionate amount of the chord axial load is carried by the chord connecting face in this region. This limit state check is omitted in Chapter 8 as such failure can be precluded by placing strict wall slenderness limits on the chord connecting face. (Hence, in Table 8-2A, \( B/t \) has an upper limit of 35 for gapped K-connections and 30 for overlapped K-connections.)

6.2 DESIGN TIPS

Whenever possible, HSS connections should be designed to be unreinforced, for reasons of both economy and aesthetics. To do so, the members need to be selected astutely at the member selection stage in order to avoid subsequent problems at the connection design stage. The design engineer responsible for member selection must perform checks on the adequacy of critical connections, to be confident with the selected members. Usually the goal is to achieve a hollow section steel structure without any stiffeners or reinforcement; clearly this would entail very careful design even with open steel sections. Some golden rules to optimize welded HSS connection design follow.

- **Select relatively stocky chord (or column) main members.** The static strength of nearly all HSS connections is enhanced by using stocky chord members, so this choice will maximize connection capacity. To achieve a high buckling resistance, there is a natural design tendency to select compression chord members that are thin-walled and have a high radius of gyration; this, however, is the classic cause of connection design problems later on. The conflicting requirements of axial load capacity, corrosion protection (a smaller surface area is obtained with stocky sections which reduces painting costs), and tube wall slenderness need to all be considered. In general, the cross-section slenderness of chord members is usually within the range \( 15c/D/t \) for round HSS and within the range \( 15c/B/t \) for square and rectangular HSS.

- **Select relatively thin branch members.** Converse to the preceding rule (where the chord members are ideally relatively thick), maximum connection efficiency will typically be realized with relatively thin branch members. The branches, however, still have an axial force requirement, so this implies using branches with large outside dimensions. Thus, a good connection design strategy is to make \( t_b/t \) as low as possible and \( B_b/B \) as high as possible, but one should still try to sit the branch on the “flat” of the chord member with rectangular HSS chord members. With \( B_b < B - 4t \), fillet welding of the branch is usually possible, and difficult, expensive flare-groove welds (arising when \( B_b = B \)) can be avoided.

- **Consider using gapped K-connections.** With truss-type construction involving K- or N-connections, gapped connections are easier and less expensive to fabricate than overlapped connections, so gapped connections are much more popular with fabricators. This is particularly the case with round-to-round HSS welded connections, where branch member ends require complex profiling and the fit-up of members requires special attention. On the other hand, overlapped K-connections—relative to gapped K-connections—generally do have a higher static (and even fatigue) strength and produce a stiffer truss with reduced truss deflections.

---

*Fig. 6-6. Local yielding of chord due to uneven load distribution.*
Chapter 7
Line Loads and Concentrated Forces on HSS

7.1 SCOPE AND BASIS

The scope of this chapter follows AISC Specification Section K1 and is intended for local “line loads” applied to the face of an HSS or to the end of an HSS member via a cap plate. The line loads are typically applied by a welded plate, oriented either longitudinal or transverse to the main HSS member axis, with the plate generally loaded in axial tension or compression. Strong-axis bending moments applied to a plate-to-round HSS connection are also covered, as is the case of shear loading on a longitudinal plate-to-HSS connection. Aside from the cap-plate case, these local loads are assumed to be applied away from the ends of an HSS member; hence, the concentrated load can be dispersed to either side of the connection. If the local line load (transverse or longitudinal) occurs near the end of the member, then it is assumed that the member end would be capped, thus regaining a similar connection strength. Wide-flange beam-to-HSS column directly welded moment connections are also indirectly covered by the scope of this chapter. The flexural capacity of such connections can be determined by ignoring the beam web and considering the beam flanges as a pair of transverse plates welded to the HSS. The moment capacity of the connection—which would be semi-rigid or partially restrained (PR)—can then be computed by multiplying the capacity of one plate-to-HSS connection by the lever arm between the centroids of the two flanges (the beam depth minus one flange thickness). When plates are oriented longitudinal to the HSS member axis, it is assumed that the plate is aligned with the HSS member axis. However, if a longitudinal plate is slightly offset from the centerline—perhaps so that a beam or diagonal bracing member centerline can coincide with the HSS column centerline—the difference in connection capacity is small and can be ignored.

Research on plate connections dates back to the 1960s (Rolloos, 1969), with numerous studies specifically applied to HSS (Kurobane, 1981; Wardenier et al., 1981; Davies and Packer, 1982; Makino et al., 1991; Cao et al., 1998; Kosteski and Packer, 2003). These form the basis of the design recommendations in this chapter; most equations (after application of appropriate resistance or safety factors) conform to CIDECT Design Guides 1 (Wardenier et al., 1991) or 3 (Packer et al., 1992) with updates in accordance with CIDECT Design Guide 9 (Kurobane et al., 2004). The latter includes revisions for longitudinal plate-to-rectangular HSS connections (AISC Specification Equation K1-9) based on extensive experimental and numerical studies reported in Kosteski and Packer (2003). Still further revisions to the plate-to-round HSS design criteria have again been recently recommended (Wardenier et al., 2008).

7.2 LIMIT STATES

Many plate-to-HSS welded connections are extremely flexible. As shown in Figure 7-1, a concentric longitudinal plate welded to a wide flange section is inherently stiffened by the presence of the web immediately behind the plate applied load. The application of the same connection practice to an HSS member results in flexure of the connecting HSS face, because the plate force is transmitted to the two HSS webs remote from the point of load application. As a consequence, the limit state of HSS plastification is a failure mode that must be commonly checked, and this serves as a control on connection deformations. The design recommendations herein make no distinction between branch plate loading in axial tension and compression. For rectangular HSS connections, there is little difference in the connection behavior—and hence the design capacity—as the connecting HSS face behaves as a laterally loaded flat plate. For plate-to-round HSS connections, however, the ultimate connection strength is usually higher under plate tension loading than under plate compression loading. For tension loading, excessive connection deformations or premature cracking may be present, well before the connection ultimate load, so—following the example of round-to-round HSS welded T-, Y- and cross-connections (see Chapter 8)—the same design capacity is conservatively used for both plate axial tension and compression. For rectangular HSS connections, there is little difference in the connection behavior—and hence the design capacity—as the connecting HSS face behaves as a laterally loaded flat plate. For plate-to-round HSS connections, however, the ultimate connection strength is usually higher under plate tension loading than under plate compression loading. For tension loading, excessive connection deformations or premature cracking may be present, well before the connection ultimate load, so—following the example of round-to-round HSS welded T-, Y- and cross-connections (see Chapter 8)—the same design capacity is conservatively used for both plate axial tension and compression.
tension and compression load cases. Hence, because deformation limits frequently control the nominal strengths of these connections, the design capacity is often considerably less than the ultimate capacity that is recorded in laboratory tests. For diagonal bracing connections to HSS columns, where the axial load is transferred to a longitudinal gusset plate, the connection capacity can be checked under the bracing force component normal to the column axis (using Table 7-1 or Table 7-2), as this will govern.

Because the available strength of a longitudinal plate-to-HSS connection is rather low, there are instances in which the connection may need to be locally reinforced (e.g., in hanger connections, high loads from cables supporting roofs or bridges, etc.). A common means of reinforcement (although not favored by fabricators) is to pass the longitudinal plate through the complete cross-section, after slotting the HSS, and to weld the “through plate” to both the front and back sides of the HSS. If this is done for a rectangular HSS (Figure 7-2) the nominal strength can be taken as twice that given by AISC Specification Equation K1-9, as is also indicated in Table 7-2 (Kosteski and Pack er, 2003). Caution should be exercised before applying this principle to plate-to-round HSS through-plate connections; recent research has suggested that the nominal strength is less than twice that of the corresponding branch-plate connection.

In addition to the limit state of HSS plastification (or HSS local yielding) described previously, other pertinent limit states for plate-to-HSS connections are (1) local yielding due to uneven load distribution in the loaded plate, (2) shear yielding (punching shear) of the HSS, and (3) sidewall strength (for rectangular HSS) by various failure modes. The provisions for the limit state of sidewall crippling of rectangular HSS (one of the sidewall failure modes), Equations K1-5 and K1-6 in the AISC Specification, conform to web crippling expressions elsewhere in the AISC Specification, and not to CIDECT or International Institute of Welding (IIW) recommendations. The pertinent limit states to be checked for plate-to-HSS connections are summarized in Table 7-1 (for round HSS) and Table 7-2 (for rectangular HSS). It is important to note that a number of potential limit states can often be precluded from the connection checking procedure because the corresponding failure modes are excluded—by virtue of the connection geometry and the limits of applicability of various parameters. The limits of applicability (given in Tables 7-1A and 7-2A) generally represent the parameter range over which the design equations have been verified in experiments or by numerical simulation. In Table 7-2 (and also in AISC Specification Section K1.3b), for transverse plate connections to rectangular HSS, it is evident that there is no check for the limit state of HSS wall plastification. This is omitted because this limit state will not govern design in practical cases. However, if there is a major compression load in the HSS—such as when it is used as a column—one should be aware that this compression load in the main member has a negative influence on the yield line plastification failure mode of the connecting HSS wall (via a $Q_f$ factor). In such a case, the designer can utilize guidance in CIDECT Design Guide 9 (Kurobane et al., 2004).
### Table 7-1. Nominal Strengths of Plate-to-Round HSS Connections

<table>
<thead>
<tr>
<th>Connection Type</th>
<th>Connection Nominal Strength*</th>
<th>Plate Bending</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Transverse-Plate T- and Cross-Connections</strong></td>
<td>Limit State: HSS Local Yielding Plate Axial Load</td>
<td>In-Plane</td>
</tr>
<tr>
<td></td>
<td>$R_n = F_t t^2 \left( \frac{5.5}{1 - 0.81 \frac{B_r}{D}} \right) Q_t$ (K1-1)</td>
<td>$M_a = 0.5 B_r R_n$</td>
</tr>
<tr>
<td></td>
<td>$\phi = 0.90$ (LRFD) $\Omega = 1.67$ (ASD)</td>
<td></td>
</tr>
<tr>
<td><strong>Longitudinal-Plate T-, Y- and Cross-Connections</strong></td>
<td>Limit State: HSS Plastification Plate Axial Load</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R_n \sin \theta = 5.5 F_t t^2 \left( 1 + 0.25 \frac{N}{D} \right) Q_t$ (K1-8)</td>
<td>$M_n = NR_n$</td>
</tr>
<tr>
<td></td>
<td>$\phi = 0.90$ (LRFD) $\Omega = 1.67$ (ASD)</td>
<td></td>
</tr>
<tr>
<td><strong>Longitudinal-Plate (Shear Tab) T-Connections</strong></td>
<td>Limit States: Plate Yielding and HSS Punching Shear Plate Shear Load</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t_p \leq \frac{F_{sy}}{F_{py}}$ from (K1-10)</td>
<td></td>
</tr>
<tr>
<td><strong>Cap-Plate Connections</strong></td>
<td>Limit State: Local Yielding of HSS Axial Load</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R_n = 2 F_t t \left( 5 t_p + N \right) \leq A F_t$ Spec. Comm. Sect. K1.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\phi = 1.00$ (LRFD) $\Omega = 1.50$ (ASD)</td>
<td></td>
</tr>
</tbody>
</table>

#### Functions

$Q_t = 1$ for HSS (connecting surface) in tension

$Q_t = 1.0 - 0.3U (1 + U)$ for HSS (connecting surface) in compression (K2-1)

$U = \frac{P_r + M_r}{AF_c + SF_c}$ (K2-2)

where $P_r$ and $M_r$ are determined on the side of the joint that has the lower compression stress. $P_r$ and $M_r$ refer to the required axial and flexural strength in the HSS. $P_r = P_r$ for LRFD; $P_r$ for ASD. $M_r = M_r$ for LRFD; $M_r$ for ASD.

* Equation references are to the AISC Specification.
### Table 7-1A. Limits of Applicability of Table 7-1

<table>
<thead>
<tr>
<th>Plate load angle:</th>
<th>$\theta \geq 30^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSS wall slenderness:</td>
<td>$D/t \leq 50$ for T-connections under branch plate axial load or bending (Spec. Sect. K1.3a)</td>
</tr>
<tr>
<td></td>
<td>$D/t \leq 40$ for Cross-connections under branch plate axial load or bending (Spec. Sect. K1.3a)</td>
</tr>
<tr>
<td></td>
<td>$D/t \leq 0.11E/F_y$ under branch plate shear loading (Spec. Comm. Sect. K1.5 requires a nonslender HSS wall)</td>
</tr>
<tr>
<td></td>
<td>$D/t \leq 0.11E/F_y$ for cap plate connections in compression (Spec. Comm. Sect. K1.5 requires a nonslender HSS wall)</td>
</tr>
<tr>
<td>Width ratio:</td>
<td>$0.2 &lt; B_p/D \leq 1.0$ for transverse branch plate connections (Spec. Sect. K1.3a)</td>
</tr>
<tr>
<td>Material strength:</td>
<td>$F_y \leq 52$ ksi (Spec. Sect. K1.2)</td>
</tr>
<tr>
<td>Ductility:</td>
<td>$F_y/F_s \leq 0.8$ (Spec. Sect. K1.2)</td>
</tr>
</tbody>
</table>

### Table 7-2. Nominal Strengths of Plate-to-Rectangular HSS Connections

#### Transverse Plate T- and Cross-Connections, under Plate Axial Load

<table>
<thead>
<tr>
<th>Connection Type</th>
<th>Connection Nominal Strength*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limit State: Local Yielding of Plate, for All $\beta$</td>
<td>$R_n = \frac{10}{B/t} F_y B_p \leq F_y t_p B_p$ (K1-2)</td>
</tr>
<tr>
<td>$\phi = 0.95$ (LRFD)</td>
<td>$\Omega = 1.58$ (ASD)</td>
</tr>
<tr>
<td>Limit State: HSS Shear Yielding (Punching), when $0.85B \leq B_p \leq 2t$</td>
<td>$R_n = 0.6F_y t_p \left( t_p + 2B_p \right)$ (K1-3)</td>
</tr>
<tr>
<td>$\phi = 0.95$ (LRFD)</td>
<td>$\Omega = 1.58$ (ASD)</td>
</tr>
<tr>
<td>Limit State: Local Yielding of HSS Sidewalls, when $\beta = 1.0$</td>
<td>$R_n = 2F_y t (5k + N)$ (K1-4)</td>
</tr>
<tr>
<td>$\phi = 1.00$ (LRFD)</td>
<td>$\Omega = 1.50$ (ASD)</td>
</tr>
<tr>
<td>Limit State: Local Crippling of HSS Sidewalls, when $\beta = 1.0$ and Plate is in Compression, for T-Connections</td>
<td>$R_n = 1.6t^2 \left( 1 + \frac{3N}{H - 3t} \right) \sqrt{E_f Q_y}$ (K1-5)</td>
</tr>
<tr>
<td>$\phi = 0.75$ (LRFD)</td>
<td>$\Omega = 2.00$ (ASD)</td>
</tr>
<tr>
<td>Limit State: Local Buckling of HSS Sidewalls, when $\beta = 1.0$ and Plates are in Compression, for Cross-Connections</td>
<td>$R_n = \frac{48t^2}{H - 3t} \sqrt{E_f Q_y}$ (K1-6)</td>
</tr>
<tr>
<td>$\phi = 0.90$ (LRFD)</td>
<td>$\Omega = 1.67$ (ASD)</td>
</tr>
</tbody>
</table>

#### Longitudinal Plate T-, Y- and Cross-Connections, under Plate Axial Load

<p>| Limit State: HSS Wall Plastification | $R_n \sin \theta = \frac{F_y t^2}{1 - \frac{t_p}{B}} \left( \frac{2N}{B} + 4 \sqrt{1 - \frac{t_p}{B}} \right)$ (K1-9) |
| $\phi = 1.00$ (LRFD) | $\Omega = 1.50$ (ASD) |</p>
<table>
<thead>
<tr>
<th>Connection Type</th>
<th>Connection Nominal Strength*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal Through-Plate T- and Y-Connections, under Plate Axial Load</td>
<td>Limit State: HSS Wall Plastification</td>
</tr>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td>$R_s \sin \theta = \frac{2F_t t^2}{1-\frac{t_p}{B}} \left[ \frac{2N}{B} + 4\sqrt{1-\frac{t_p}{B}} \right]$ Spec. Comm. Sect. K1.3</td>
</tr>
<tr>
<td></td>
<td>$\phi = 1.00 \text{ (LRFD)}$ $\Omega = 1.50 \text{ (ASD)}$</td>
</tr>
<tr>
<td>Longitudinal Plate (Shear Tab) T-Connections, under Plate Shear Load</td>
<td>Limit States: Plate Yielding and HSS Punching Shear</td>
</tr>
<tr>
<td><img src="image2" alt="Diagram" /></td>
<td>$t_p \leq \frac{F_t}{F_{tp}}$ from (K1-10)</td>
</tr>
<tr>
<td>Cap-Plate Connections, under Axial Load</td>
<td>Limit State: Local Yielding of Sidewalls</td>
</tr>
<tr>
<td><img src="image3" alt="Diagram" /></td>
<td>$R_b = 2F_t t \left( 5t_p + N \right)$, for $\left( 5t_p + N \right) &lt; B$ Spec. Sect. K1.6</td>
</tr>
<tr>
<td></td>
<td>$R_b = AF_t$, for $\left( 5t_p + N \right) \geq B$</td>
</tr>
<tr>
<td></td>
<td>$\phi = 1.00 \text{ (LRFD)}$ $\Omega = 1.50 \text{ (ASD)}$</td>
</tr>
<tr>
<td></td>
<td>Limit State: Local Crippling of Sidewalls, when Plate is in Compression</td>
</tr>
<tr>
<td><img src="image4" alt="Diagram" /></td>
<td>$R_b = 1.6t^2 \left[ 1 + \frac{5N}{B} \left( \frac{t}{t_p} \right)^{1.5} \right] \sqrt{EF_t \frac{t_p}{t}}$, for $\left( 5t_p + N \right)&lt;B$ (K1-12)</td>
</tr>
<tr>
<td></td>
<td>$\phi = 0.75 \text{ (LRFD)}$ $\Omega = 2.00 \text{ (ASD)}$</td>
</tr>
</tbody>
</table>
### Table 7-2. (continued)

<table>
<thead>
<tr>
<th>Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_r = 1 ) for HSS (connecting surface) in tension</td>
</tr>
<tr>
<td>( Q_r = 1.3 - 0.4 \frac{U}{\beta} \leq 1.0 ) for HSS (connecting surface) in compression, for transverse plate connections (K2-10)</td>
</tr>
<tr>
<td>( Q_r = \sqrt{1 - U^2} ) for HSS (connecting surface) in compression, for longitudinal plate and longitudinal through plate connections</td>
</tr>
<tr>
<td>( U = \frac{P_r + M_t}{AF_c + SF_t} ) (K2-12)</td>
</tr>
</tbody>
</table>

where \( P_r \) and \( M_t \) are determined on the side of the joint that has the lower compression stress. \( P_r \) and \( M_r \) refer to the required axial and flexural strength in the HSS. \( P_r = P_u \) for LRFD; \( P_a \) for ASD. \( M_r = M_u \) for LRFD; \( M_a \) for ASD.

\[
B_{wp} = \frac{10B_0}{B/t} \leq B_{fwp} \quad \text{Spec. K1.3b}
\]

\( k \) = outside corner radius of HSS \( \geq 1.5t \)

* Equation references are to the AISC Specification.

### Table 7-2A. Limits of Applicability of Table 7-2

<table>
<thead>
<tr>
<th>Limitation</th>
<th>Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate load angle:</td>
<td>( \theta \geq 30^\circ )</td>
</tr>
<tr>
<td>HSS wall slenderness:</td>
<td>( B/t ) or ( H/t \leq 35 ) for loaded wall, for transverse branch plate connections (Spec. Sect. K1.3b)</td>
</tr>
<tr>
<td></td>
<td>( B/t ) or ( H/t \leq 40 ) for loaded wall, for longitudinal branch plate and through plate connections (Spec. Sect. K1.4b)</td>
</tr>
<tr>
<td></td>
<td>((B - 3t)/t ) or ((H - 3t)/t \leq 1.40 \sqrt{E/F_y} ) for loaded wall, for branch plate shear loading (Spec. Comm. Sect. K1.6 requires a nonslender HSS wall per Spec. Sect. B4.1)</td>
</tr>
<tr>
<td>Width ratio:</td>
<td>( 0.25 \leq B_w/B \leq 1.0 ) for transverse branch plate connections (Spec. Sect. K1.3b)</td>
</tr>
<tr>
<td>Material strength:</td>
<td>( F_y \leq 52 ) ksi (Spec. Sect. K1.2)</td>
</tr>
<tr>
<td>Ductility:</td>
<td>( F_y/F_c \leq 0.8 ) (Spec. Sect. K1.2)</td>
</tr>
</tbody>
</table>
7.4 LONGITUDINAL-PLATE AND CAP-PLATE CONNECTIONS

The design method for longitudinal-plate T-connections with the plate loaded in shear, better known as “shear-tab” connections, is applicable to simple (or shear) connections between wide-flange beams and HSS columns. The recommendations herein are based on research by Sherman and Ales (1991) and Sherman (1995, 1996) that investigated a large number of simple framing connections between wide-flange beams and rectangular HSS columns. Although the beam end reaction was predominantly a shear force, a small amount of end moment was always present, as is typical in practice. Over a wide range of connections tested, only one limit state was identified for the rectangular HSS column: punching shear failure related to the small end rotation of the beam. Moreover, this only occurred when a thick shear plate was joined to a relatively thin-walled HSS. Thus, to avoid this HSS failure mode, the shear tab could be made relatively thin so that the shear tab yielded under axial tension and compression force (by treating the moment in the shear tab as a tension/compression force couple) prior to punching shear failure of the HSS connecting wall, adjacent to that local tension/compression force. Hence, in limit states or LRFD terminology:

Yielding of shear tab, per unit length of tab \( \leq \) Shear rupture of HSS wall along the tab sides, per unit length of tab

\[
\text{i.e., } \left( \phi = 0.90 \right) F_{ytp} \leq \left( \phi_p = 0.75 \right) \left( 0.6F_y \right) 2t \quad (7-1)
\]

or \( F_{ytp} \leq F_y t \)

which is implemented in the AISC Specification as Equation K1-10 and in Table 7-2 of this Design Guide. A limit of validity for this recommendation given in AISC Specification Commentary Section K1.5 requires the rectangular HSS to be nonslender; hence, in accordance with AISC Specification Table B4.1, there is an accompanying criterion that \((B - 3s)/t\) of the loaded face \( \leq 1.40(E/F_y)^{0.5}\). In AISC Specification Commentary Section K1.5, as well as in Table 7-1 of this Design Guide, an extrapolation of Equation K1-10 has been made for shear tabs to round HSS columns—again with the provision that the round HSS cross-section is nonslender (i.e., \(D/t \leq 0.11E/F_y\)). Equation 7-1 or AISC Specification Equation K1-10 thus requires that the shear tab be kept relatively thin. Single-plate and single-angle, shear-tab connections were also determined to be the least expensive type of simple shear connection by Sherman (1995). Double-angle and fillet-welded tee connections were more expensive, with the most expensive shear connections being the through-plate and flare-bevel-welded tee connections.

The design method for cap-plate connections to the ends of HSS members, where the axial force in a plate is transferred to the HSS via a cap plate (or via the flange of a tee-stub), recognizes that shear lag will be present in the HSS if some of the cross-section is not loaded. A conservative distribution slope of 2.5:1 is assumed from each face of the tee web (Wardenier et al., 1991; Kitipornchai and Traves, 1989), which produces a dispersed load width of \((5t_p + N)\) as shown in Figure 7-3, leading to the recommendation in Table 7-1. This load model is analogous to that used elsewhere in the AISC Specification, in Equation J10-2. It can be seen in Figure 7-3 that the round HSS capacity \((F_yA)\) is fully utilized once the dispersed load width of \((5t_p + N)\) \(\geq D\). This will be achieved only with a very thick cap plate.

The same load dispersion model is used for cap-plate connections to square and rectangular HSS, leading to Equation K1-11 in the AISC Specification and in Table 7-2 of this Design Guide. In that case, when \((5t_p + N) < B\), and assuming that the cap plate is rectangular with its long dimension parallel to the \(H\) dimension of the HSS, just two side-walls of the HSS will be loaded, and the connection nominal strength for the limit state of local yielding will be \(R_n = 2F_y t (5t_p + N)\). If \((5t_p + N) \geq B\), the whole HSS will be loaded and no shear lag will be present. This is confirmed in Table 7-2.

The AISC Specification also includes another limit state to be checked if the plate is in compression—local crippling of the HSS side-walls (Equation K1-12). This has been carried over from AISC Specification Equation J10-5a, which applies to a beam web loaded by a concentrated force. This check (Equation K1-12), for HSS cap-plate connections, is not discussed in other prominent international design recommendations (Wardenier et al., 1991; Packer et al., 1992; Packer and Henderson, 1997; Wardenier et al., 2008).

![Fig. 7-3. Load dispersion through a cap-plate-to-round HSS connection.](image-url)
Furthermore, AISC Specification Equation K1-12 can clearly be applied to two rectangular HSS walls when \((5t_p + N) < B\), but if \((5t_p + N) \geq B\), the application to all four walls is questionable. If one assumes that \((5t_p + N) < B\), Equation K1-11 (local yielding) would govern over Equation K1-12 (wall local crippling) in the AISC Specification, for one wall, if (with application of the relevant resistance factors):

\[
1.00 F_t t_p (5t_p + N) \leq 0.75 (0.8) t_p^2 \left[ 1 + \frac{6N}{B} \left( \frac{t}{t_p} \right) \right] \left( \frac{E}{t} \right) \frac{t_p}{t} \quad (7-2)
\]

Assuming that \(E = 29,000\) ksi and \(f_y = 50\) ksi:

\[
(5t_p + N) \leq 14.45 \sqrt{t/t_p} + 86.7 \left( \frac{N}{B} \right) \left( \frac{t^2}{t_p} \right) \quad (7-3)
\]

Web crippling would only be critical over web yielding for very thin HSS walls, so assume a punitive value of \(B/t = 50\).

Thus:

\[
\left(5t_p + N\right) \leq 14.45 \sqrt{t/t_p} + 1.73N \left( \frac{t}{t_p} \right) \quad (7-4)
\]

It can be seen that this inequality holds (and hence web crippling does not govern) for \(0.05B \leq N \leq 0.2B\) over the range of \(1.0 \leq t_p/t \leq 4.0\). Thus, for practical connections AISC Specification Equation K1-12 is never likely to govern. Equation K1-12 becomes critical over Equation K1-11 for large bearing lengths, \(N\), combined with thick cap plates, but for these combinations, shear lag is negated and all four walls of the HSS will participate, thus rendering Equation K1-12 invalid.

### 7.5 Connection Design Examples

**Example 7.1—Branch/Through-Plate Connection with Rectangular HSS**

**Given:**

The welded HSS connection shown in Figure 7-4 is subject to the plate axial tension force indicated produced by a diagonal bracing member. The column is subject to axial compression dead and live loads of \(P_D = 20.0\) kips and \(P_L = 60.0\) kips. Determine if the connection is adequate as a branch-plate connection or if a through-plate connection is required (as illustrated later). Note that the connection is checked under the action of the plate force component normal to the column, as this has the dominant effect.

![Fig. 7-4. Through-plate connection with rectangular HSS.](image-url)
From AISC *Manual* Tables 2-3 and 2-4, the material properties are as follows:

HSS8×8×¼
ASTM A500 Grade B
\( F_y = 46 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

PL\( \frac{1}{4} \times 8 \)
ASTM A36
\( F_{yp} = 36 \text{ ksi} \)
\( F_{up} = 58 \text{ ksi} \)

From AISC *Manual* Table 1-12, the HSS geometric properties are as follows:

HSS8×8×¼
\( B = 8.00 \text{ in.} \)
\( H = 8.00 \text{ in.} \)
\( t = 0.233 \text{ in.} \)
\( A = 7.10 \text{ in.}^2 \)

PL\( \frac{1}{4} \times 8 \)
\( t_p = 0.250 \text{ in.} \)
\( A_g = 2.00 \text{ in.}^2 \)

**Solution:**

*Limits of applicability of AISC Specification Section K1 (also see Table 7-2A)*

From AISC *Specification* Section K1.2, check the limits of applicability.

\( F_y = 46 \text{ ksi} \leq 52 \text{ ksi} \quad \text{O.K.} \)

\( F_y / F_u = 0.793 \leq 0.8 \quad \text{O.K.} \)

From AISC *Specification* Section K1.4b, check additional criterion for rectangular HSS.

\( B/t = H/t \)

\( = 34.3 \leq 40 \quad \text{O.K.} \)

*Required strength (expressed as a force in the plate)*

From Chapter 2 of ASCE 7, the required strength of the connection is:

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
R_u = 1.6 \left( 25.0 \text{ kips} \right) & \\
= 40.0 \text{ kips} & R_u = 25.0 \text{ kips} \\
\hline
\end{array}
\]

*HSS chord wall plastification, assuming a branch-plate connection*

From AISC *Specification* Section K1.4b, determine the nominal strength of the HSS for the limit state of chord wall plastification assuming a branch-plate connection.

\[
R_u \sin \theta = \frac{F_y t^2}{1 - \frac{t_p}{B}} \left[ \frac{2N}{B} + 4 \sqrt{1 - \frac{t_p}{B} Q_f} \right]
\]

(from Spec. Eq. K1-9 and Table 7-2)

where

\( Q_f = \sqrt{1 - U^2} \) for HSS (column) in compression

\[
U = \left| \frac{P_r}{AF_c + SF_c} \right|
\]

(Spec. Eq. K2-12 and Table 7-2)
From Chapter 2 of ASCE 7, the required strength, \( P_r \), in the HSS chord is determined as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_r = P_r = 1.2 \left( 20.0 \text{ kips} \right) + 1.6 \left( 77.7 \text{ kips} \right) )</td>
<td>( P_r = P_r = 20.0 \text{ kips} + 77.7 \text{ kips} )</td>
</tr>
<tr>
<td>= 148 kips in column on side of joint with higher compression stress</td>
<td>= 97.7 kips in column on side of joint with higher compression stress</td>
</tr>
<tr>
<td>( F_c = F_c )</td>
<td>( F_c = 0.6 F_c )</td>
</tr>
<tr>
<td>( U = \left[ \frac{148 \text{ kips}}{71.0 \text{ in.}^2 \left( 46 \text{ ksi} \right)} \right] )</td>
<td>( U = \left[ \frac{97.7 \text{ kips}}{71.0 \text{ in.}^2 \left( 27.6 \text{ ksi} \right)} \right] )</td>
</tr>
<tr>
<td>= 0.453</td>
<td>= 0.499</td>
</tr>
<tr>
<td>( Q_f = \sqrt{1 - 0.45^2} )</td>
<td>( Q_f = \sqrt{1 - 0.499^2} )</td>
</tr>
<tr>
<td>= 0.892</td>
<td>= 0.867</td>
</tr>
</tbody>
</table>

The available strength for the limit state of HSS chord wall plastification is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_n = 46 \text{ ksi} \left( 0.233 \text{ in.} \right)^2 \left[ \frac{2 \left( 8.00 \text{ in.} / \sin 45^\circ \right)}{8.00 \text{ in.}} \right] ) ( + 4 \sqrt{1 - \frac{0.250 \text{ in.}}{8.00 \text{ in.}} \left( 0.892 \right)} ) / sin 45°</td>
<td>( R_n = 46 \text{ ksi} \left( 0.233 \text{ in.} \right)^2 \left[ \frac{2 \left( 8.00 \text{ in.} / \sin 45^\circ \right)}{8.00 \text{ in.}} \right] ) ( + 4 \sqrt{1 - \frac{0.250 \text{ in.}}{8.00 \text{ in.}} \left( 0.867 \right)} ) / sin 45°</td>
</tr>
<tr>
<td>= 23.1 kips</td>
<td>= 22.8 kips</td>
</tr>
<tr>
<td>( \phi R_n = 1.00 \left( 23.1 \text{ kips} \right) )</td>
<td>( \phi R_n = 22.8 \text{ kips} / 1.50 )</td>
</tr>
<tr>
<td>= 23.1 kips</td>
<td>= 15.2 kips</td>
</tr>
<tr>
<td>23.1 kips &lt; 40.0 kips</td>
<td>15.2 kips &lt; 25.0 kips</td>
</tr>
</tbody>
</table>

Because the available strength for the limit state of HSS chord wall plastification, treating the connection as a branch-plate connection, is not adequate, check this limit state treating the connection as a through-plate connection.

**HSS wall plastification, assuming a through-plate connection**

Using Table 7-2 and AISC Specification Commentary K1.3, determine the nominal strength of the HSS for the limit state of chord wall plastification assuming a through-plate connection.

\[
R_{n, \sin \theta} = \frac{2 F_c t_r^2 \left( \frac{2N}{B} + 4 \sqrt{1 - \frac{t_r}{B} Q_f} \right)}{1 - \frac{t_r}{B}}
\]

This gives a nominal connection strength that is double that of a branch-plate connection (AISC Specification Equation K1-9), hence \( \phi R_n = 46.2 \text{ kips} > 40.0 \text{ kips} \) and \( R_n / \Omega = 30.4 \text{ kips} > 25.0 \text{ kips} \), so the connection will be adequate.
Tensile yielding in the gross section of the plate

From AISC Specification Section D2, the nominal strength for the limit state of tension yielding on the plate is:

\[ P_n = F_y A_t \]

\[ = 36 \text{ ksi} \left( 2.00 \text{ in.}^2 \right) \]

\[ = 72.0 \text{ kips} \]  

(Spec. Eq. D2-1)

The available strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \( \phi P_n = 0.90 \left( 72.0 \text{ kips} \right) \) \= 64.8 kips | \( \frac{P_n}{\Omega} = \frac{72.0 \text{ kips}}{1.67} \)
| \[ 64.8 \text{ kips} > 40.0 \text{ kips} \text{ o.k.} \] | \[ = 43.1 \text{ kips} \text{ o.k.} \] |

Example 7.2—W-Shape Beam-to-Round HSS Column Moment Connection

Given:

A W18\times35 beam is profiled at its end and directly welded to an HSS8.625\times0.500 column. Determine the adequacy of the connection under the loads indicated in Figure 7-5. Note that the beam will be treated as a pair of transverse plates. The critical plate connection will be the lower plate, as the column section adjoining the plate develops compressive stresses due to both axial force and bending. These compressive stresses cumulatively affect the \( U \) factor. The moment capacity of the entire beam-to-column connection is then based on the capacity of the lower (compression-flange) plate connection multiplied by the distance between the beam flange centers.

Fig. 7-5. Beam-to-round HSS moment connection.
From AISC Manual Table 2-3, the material properties are as follows:

- **HSS 8.625×0.500**
  - ASTM A500 Grade B
  - \( F_y = 42 \text{ ksi} \)
  - \( F_u = 58 \text{ ksi} \)

- **W18×35**
  - ASTM A992
  - \( F_{yp} = 50 \text{ ksi} \)
  - \( F_{up} = 65 \text{ ksi} \)

From AISC Manual Tables 1-13 and 1-1, the geometric properties are as follows:

- **HSS 8.625×0.500**
  - \( D = 8.625 \text{ in.} \)
  - \( t = 0.465 \text{ in.} \)
  - \( A = 11.9 \text{ in}^2 \)
  - \( S = 23.1 \text{ in}^3 \)

- **W18×35**
  - \( d = 17.7 \text{ in.} \)
  - \( b_f = 6.00 \text{ in.} \)
  - \( t_f = 0.425 \text{ in.} \)

**Solution:**

*Limits of applicability as a transverse-plate connection*

From AISC Specification Section K1.2 and Table 7-1A, check the limits of applicability.

- \( F_y = 42 \text{ ksi} \leq 52 \text{ ksi} \) **o.k.**
- \( F_y / F_u = 0.724 \leq 0.8 \) **o.k.**

From AISC Specification Section K1.3a, check additional criterion for round HSS.

- \( D/t = 18.5 \leq 50 \) **o.k.**
- \( 0.2 < B_f / D = 0.696 \leq 1.0 \) **o.k.**

*Required flexural strength (expressed as a moment in the beam)*

From Chapter 2 of ASCE 7, the required flexural strength is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_u = 1.2(100 \text{ kip-in.}) + 1.6(300 \text{ kip-in.}) )</td>
<td>( M_u = 100 \text{ kip-in.} + 300 \text{ kip-in.} )</td>
<td></td>
</tr>
<tr>
<td>= 600 kip-in.</td>
<td>= 400 kip-in.</td>
<td></td>
</tr>
</tbody>
</table>

**HSS (column) local yielding, as a transverse-plate connection**

From AISC Specification Section K1.3a, the nominal strength of the HSS for the limit state of local yielding is:

\[
R_n = F_y t^2 \left( \frac{5.5}{1 - 0.81 \frac{B_f}{D}} \right) Q_f
\]

\( (Spec. \text{ Eq. K1-1 amd Table 7-1}) \)
where
\[ Q_f = 1.0 - 0.3U (1 + U) \] for HSS (column) surface in compression \hspace{1cm} \text{(Spec. Eq. K2-1 and Table 7-1)}

\[ U = \frac{P_r + M_r}{AF_c + SF_c} \] on the side with the lower compression stress \hspace{1cm} \text{(Spec. Eq. K2-2 and Table 7-1)}

From Chapter 2 of ASCE 7, the required strengths, \( P_r \) and \( M_r \) in the HSS are determined as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_r = P_a )</td>
<td>( P_r = P_a )</td>
</tr>
<tr>
<td>[ = 1.2 (50.0 \text{ kips}) + 1.6 (150 \text{ kips}) ] 300 kips</td>
<td>[ = 50.0 \text{ kips} + 150 \text{ kips} ] 200 kips</td>
</tr>
<tr>
<td>( M_r = M_a )</td>
<td>( M_r = M_a )</td>
</tr>
<tr>
<td>[ = 1.2 (50.0 \text{ kip-in.}) + 1.6 (150 \text{ kip-in.}) ] 300 kip-in.</td>
<td>[ = 50.0 \text{ kip-in.} + 150 \text{ kip-in.} ] 200 kip-in.</td>
</tr>
<tr>
<td>( F_c = F_i )</td>
<td>( F_c = 0.6F_i )</td>
</tr>
<tr>
<td>= 42 ksi</td>
<td>= 25.2 ksi</td>
</tr>
<tr>
<td>( U = \frac{300 \text{ kips}}{11.9 \text{ in.}^2 (42 \text{ ksi})} + \frac{300 \text{ kip-in.}}{23.1 \text{ in.}^3 (42 \text{ ksi})} ) 0.909</td>
<td>[ = \frac{200 \text{ kips}}{11.9 \text{ in.}^2 (25.2 \text{ ksi})} + \frac{200 \text{ kip-in.}}{23.1 \text{ in.}^3 (25.2 \text{ ksi})} ] 1.01</td>
</tr>
<tr>
<td>( Q_f = 1.0 - 0.3 (0.909)(1 + 0.909) ) 0.479</td>
<td>( Q_f = 1.0 - 0.3 (1.01)(1 + 1.01) ) 0.391</td>
</tr>
</tbody>
</table>

The available axial and flexural strength for the limit state of HSS local yielding are determined as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_a = 42 \text{ ksi} (0.465 \text{ in.})^2 \times \left[ \frac{5.5}{1 - 0.81} \left(\frac{6.00 \text{ in.}}{8.625 \text{ in.}}\right) \right] (0.479) ) 54.8 kips</td>
<td>( R_a = 42 \text{ ksi} (0.465 \text{ in.})^2 \times \left[ \frac{5.5}{1 - 0.81} \left(\frac{6.00 \text{ in.}}{8.625 \text{ in.}}\right) \right] (0.391) ) 44.7 kips</td>
</tr>
<tr>
<td>( \phi R_a = 0.90 \times 54.8 \text{ kips} ) 49.3 kips</td>
<td>( \phi R_a = 0.90 \times 44.7 \text{ kips} ) 40.3 kips</td>
</tr>
<tr>
<td>( \phi M_a = 49.3 \text{ kips} \left(17.7 \text{ in.} - 0.425 \text{ in.}\right) ) 852 kip-in. &gt; 600 kip-in. \textbf{O.K.}</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 8
HSS-to-HSS Truss Connections

8.1 SCOPE AND BASIS

The scope of this chapter follows AISC Specification Section K2 and is limited to planar, truss-type connections between HSS (or box members) directly welded to one another, in the form of T-, Y-, cross-, K- (or N-) gapped or overlapped connections. Round HSS-to-round HSS and rectangular HSS-to-rectangular HSS are the only combinations of HSS shapes considered. Pertinent design criteria for round HSS-to-HSS truss connections are succinctly tabulated (Table 8-1), followed by two design examples on such connections. Similarly, pertinent design criteria for rectangular (which includes square) HSS-to-HSS truss connections are tabulated (Table 8-2), followed by three design examples on such connections. For connection configurations beyond the scope of this chapter such as multi-planar connections, connections with partially or fully flattened branch member ends, double chord connections, connections with a branch member that is offset so that its centerline does not intersect with the centerline of the chord, connections with round branch members joined to a square or rectangular HSS chord member, or even HSS branch members joined to a W-section chord member, one can refer to other authoritative design guidance. Such publications include IIW (1989), CIDECT Design Guide No.1 (Wardenier et al., 1991), CIDECT Design Guide No.3 (Packer et al., 1992), the CISC Design Guide (Packer and Henderson, 1997), Marshall (1992), and AWS D1.1 (AWS, 2006).

The design criteria in this chapter and AISC Specification Section K2 are based on potential limit states that have been reported in international research on HSS, much of which has been sponsored and synthesized by the International Committee for the Development and Study of Tubular Construction (CIDECT) since the 1960s. This work has also received critical review by the International Institute of Welding (IIW) Subcommission XV-E on welded joints in tubular structures. The HSS connection design provisions herein are generally in accord with the latest edition of the design recommendations by this Subcommission (IIW, 1989). Some minor modifications to the IIW recommendations—for some limit states—have been made by the adoption of the formulas for the same limit states elsewhere in the AISC Specification. The IIW connection design recommendations referred to above have also been implemented and supplemented in later design guides by CIDECT (Wardenier et al., 1991; Packer et al., 1992), by CISC (Packer and Henderson, 1997), and in Eurocode 3 (CEN, 2005). A large amount of connection research data generated by CIDECT research programs up to the early 1980s is summarized in CIDECT Monograph No. 6 (Giddings and Wardenier, 1986) and by Wardenier (1982). Further information on CIDECT publications and reports can be obtained from its website: www.cidect.com.

8.2 NOTATION AND LIMIT STATES

Some common notation associated with welded HSS connections that is used in this chapter is shown in Figure 8-1. The design of welded HSS connections is based on potential limit states that may arise for a particular connection.
of applicability of various parameters. The limits of applicability (given in Tables 8-1A and 8-2A) generally represent the parameter range over which the design equations have been verified in experiments.

### 8.3 CONNECTION CLASSIFICATION

HSS-to-HSS truss connections consist of one or more branch members that are directly welded to a continuous chord that passes through the connection. The classification of these geometry and loading, which in turn represent possible failure modes that may occur within prescribed limits of applicability. Typical failure modes for truss-type connections, illustrated for a rectangular HSS gapped K-connection, are given in Figure 8-2. These limit states are summarized in Table 8-1 (for round HSS) and Table 8-2 (for rectangular HSS). It is important to note that a number of potential limit states can often be omitted from the connection checking procedure because the corresponding failure modes are excluded, by virtue of the connection geometry and the limits of applicability of various parameters. The limits of applicability (given in Tables 8-1A and 8-2A) generally represent the parameter range over which the design equations have been verified in experiments.

![Fig. 8-2. Typical limit states for HSS-to-HSS truss connections.](image-url)
connections as K- (which includes N-), Y- (which includes T-), or cross- (also known as X-) connections is based on the method of force transfer in the connection, not on the physical appearance of the connection. Thus:

a. When the punching load \( (P, \sin \theta) \) in a branch member is equilibrated by beam shear in the chord member, the connection is classified as a T-connection when the branch is perpendicular to the chord and a Y-connection otherwise.

b. When the punching load \( (P, \sin \theta) \) in a branch member is essentially equilibrated (within 20\%) by loads in other branch member(s) on the same side of the connection, the connection is classified as a K-connection. The relevant gap is between the primary branch members whose loads equilibrate.

c. When the punching load \( (P, \sin \theta) \) is transmitted through the chord member and is equilibrated by branch member(s) on the opposite side, the connection is classified as a cross-connection.

Examples of these connection classifications are shown in Figure 8-3. When branch members transmit part of their load as K-connections and part of their load as T-, Y- or cross-connections, the adequacy of each branch is determined by linear interaction of the proportion of the branch load involved in each type of load transfer. One K-connection, in Figure 8-3(b), illustrates that the branch force components normal to the chord member may differ by as much as 20\% and still be deemed to exhibit K-connection behavior. This is to accommodate slight variations in branch member forces along a typical truss, caused by a series of panel point loads. The N-connection in Figure 8-3(c), however, has a ratio of branch force components normal to the chord member of 2:1. In the case shown in Figure 8-3(b), the connection needs to be analyzed as both a “pure” K-connection (with balanced branch forces) and a cross- (or X-) connection (because the remainder of the diagonal branch load is being transferred through the connection), as shown in Figure 8-4. For the diagonal tension branch in that connection, the following check is made:

\[
0.5 P, \sin \theta \leq \text{K-connection available strength} + \frac{0.5 P, \sin \theta}{\text{cross-connection available strength}} \leq 1.0
\]

Example 8.5 later in this chapter gives a demonstration of how design calculations are performed for such a connection. If the gap size in a gapped K- (or N-) connection [e.g., Figure 8-3(a)] becomes large and exceeds the value permitted by the eccentricity limit (in Table 8-1A or Table 8-2A), then the K-connection should be treated as two independent Y-connections. In cross-connections, such as Figure 8-3(h), where the branches are close together or overlapping, the combined “footprint” of the two branches can be taken as the loaded area on the chord member. Example 8.3 later in this chapter demonstrates this technique for determining the bearing length on the chord. In K-connections such as Figure 8-3(d), where a branch has very little or no loading, the connection can be treated as a Y-connection, as shown. If Figure 8-3(i) has gapped K-connections, it can be seen that there is a high shear force on the chord cross-section taken through the gap region. In this case the chord shear capacity should be checked (see Table 8-2 for rectangular HSS chords). If the KK-connection shown in Figure 8-3(i) has the forces in the lower K-connection reversed in sense (direction), there would be a significant equilibrating compression force in the chord member. In such cases, this total compression force should be used in computing \( Q_i \) (see Tables 8-1 and 8-2) for each K-connection.

The same effective weld size should be maintained all around the attached branch, except for the “hidden weld” in HSS-to-HSS partially overlapped K- or N-connections, which may be left unwelded (usually just tacked) provided the force components of the two branches normal to the chord do not differ by more than 20\%. (The hidden weld refers to the weld along the “hidden toe” of the overlapped branch, which is hidden in the final connection by the overlapping branch. This is particularly an issue with square/rectangular partially overlapped HSS-to-HSS connections where the typical fabrication procedure is to tack all the branches into place then perform final welding afterwards.)

### 8.4 TRUSS MODELING AND MEMBER DESIGN

This chapter, and, similarly, AISC Specification Section K2, presumes that the branches are loaded only by axial forces. Within the constraints of the limits of applicability in Tables 8-1A and 8-2A, the welded connections within a truss will be semi-rigid (or partially rigid) and branch member stiffnesses will be considerably less than the chord member stiffness. As a result, the actual bending moments in branch members will be very low, and less than would be reflected by a rigid-frame analysis. Consequently, the recommended methods for performing analysis of planar welded HSS trusses are:

1. Pin-jointed analysis.

2. Analysis using web members pin-connected to continuous chord members, as shown in Figure 8-5. The extremely stiff members shown should have section properties greater than the chord member and a length equal to the noding eccentricity, \( e \) (see Figure 8-1). If chord member loads are applied off the joints (panel points), this method of analysis provides realistic bending moment distributions for chord member design.
Fig. 8-3. Examples of HSS connection classification.

Fig. 8-4. Checking of a K-connection with imbalanced branch member loads.
By both of the preceding analysis methods the web members will have zero bending moments (unless there are applied loads directly to the web members). If a rigid-frame analysis is carried out, it is recommended that the bending moments generated in the web members be ignored. It is worth noting that the member axial forces produced are generally very similar, by either of the three analysis methods (rigid-frame analysis being termed method 3.).

When selecting members for an HSS truss it must be reiterated that minimum weight does not equal minimum cost. The number of different member sizes should be small, and the number of connections minimized, to reduce the fabrication cost. For fewer connections, Warren trusses can be a good choice. Because one is generally required to produce an HSS truss design without stiffened connections, it is common to find that member selection is governed by connection design criteria. The most common cause for HSS connection design problems is the selection of chord members with a high $D/t$ ratio or high $B/t$ ratio, particularly for the compression chord, where a high buckling resistance is sought and hence a member with a high radius of gyration may have been initially chosen. Optimal chord members that produce efficient connections are relatively stocky: typical choices for round HSS chord members are in the range of $15 \leq D/t \leq 30$, and typical choices for square HSS chord members are in the range $15 \leq B/t \leq 25$. On the other hand, branch members should—in general—be chosen to have a high $D/t$ or $B/t$, but within the limits permitted by Tables 8-1A and 8-2A. However, branches should preferably still sit on the “flat” of the chord member, for rectangular HSS trusses, to simplify welding procedures. Thus, branch members are generally selected to be relatively wide and thin, whereas chord members are generally selected to be stocky and thick, resulting in $t_c \leq t$. Still, reference to Table 8-1A shows that the requirement for round HSS compression branches ($D_b/t_b \leq 0.05E/F_{yb}$) is well below the round HSS noncompact section limit (see AISC Specification Table B4.1). Similarly, reference to Table 8-2A shows that the requirement for rectangular HSS compression branches in overlapped K-connections ($B_b/t_b$ and $H_b/t_b \leq 1.1(E/F_{yb})^{0.5}$) is even below the rectangular HSS compact section limit (see AISC Specification Table B4.1).

The package of connection design criteria in Tables 8-1 and 8-2, in conjunction with the limits of applicability in Tables 8-1A and 8-2A, permits the use of effective lengths, $KL$, less than $L$ when designing planar-welded HSS truss compression members. For compression chords, $KL = 0.9L$ can be adopted, where $L$ is the distance between chord panel points (for buckling in the plane of the truss) or $L$ is the distance between points of lateral support for the chord (for buckling perpendicular to the truss). For compression web members, the use of $KL = 0.75L$ is permissible, where $L$ is the distance between member nodes (CIDECT, 1980; Mouty, 1981; Rondal et al., 1992).

Long, laterally unsupported compression chords can exist in U-frame [or “half-through” (pony) truss] pedestrian bridges and in roof trusses subjected to large wind uplift (the bottom chord). The effective length of such laterally unsupported truss chords can be considerably less than the unsupported length, and design guidance is provided by Packer and Henderson (1997) and Galambos (1998).

Fig. 8-5. Modeling assumption using web members pin-connected to continuous chord members.
### Table 8-1. Nominal Strengths of Round HSS-to-HSS Truss Connections

<table>
<thead>
<tr>
<th>Connection Type</th>
<th>Connection Nominal Axial Strength*</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Check</td>
<td>Limit State: Shear Yielding (Punching)</td>
</tr>
<tr>
<td>For T-, Y-, Cross- and K-Connections with Gap, when $D_{b (tens/comp)} &lt; (D - 2t)$</td>
<td>$P_n = 0.6 F_t t D_n \left( \frac{1 + \sin \theta}{2 \sin^2 \theta} \right)$ and $\phi = 0.95$ (LRFD) $\Omega = 1.58$ (ASD)</td>
</tr>
<tr>
<td>T- and Y-Connections</td>
<td>Limit State: Chord Plastification</td>
</tr>
<tr>
<td>$P_n \sin \theta = F_t t^2 \left( 3.1 + 15.6 \beta^2 \right) \gamma^{0.2} Q_i$</td>
<td>$\phi = 0.90$ (LRFD) $\Omega = 1.67$ (ASD)</td>
</tr>
<tr>
<td>Cross-Connections</td>
<td>Limit State: Chord Plastification</td>
</tr>
<tr>
<td>$P_n \sin \theta = F_t t^2 \left( \frac{5.7}{1 - 0.81 \beta} \right) Q_i$</td>
<td>$\phi = 0.90$ (LRFD) $\Omega = 1.67$ (ASD)</td>
</tr>
<tr>
<td>K-Connections with Gap or Overlap</td>
<td>Limit State: Chord Plastification</td>
</tr>
<tr>
<td>$(P_n \sin \theta)<em>{compression \ branch} = F_t t^2 \left( 2.0 + 11.33 \frac{D</em>{b \ (comp)}}{D} \right) Q_i$</td>
<td>$\phi = 0.90$ (LRFD) $\Omega = 1.67$ (ASD)</td>
</tr>
<tr>
<td>$(P_n \sin \theta)<em>{tension \ branch} = (P_n \sin \theta)</em>{compression \ branch}$</td>
<td></td>
</tr>
</tbody>
</table>

**Functions**

- $Q_i = 1$ for chord (connecting surface) in tension
- $Q_i = 1.0 - 0.3 U (1 + U)$ for chord (connecting surface) in compression

$$U = \frac{P_r}{AF_c + SF_c}$$

where $P_r$ and $M_r$ are determined on the side of the joint that has the lower compression stress. $P_r$ and $M_r$ refer to the required axial and flexural strength in the HSS. $P_r = P_{r_u}$ for LRFD; $P_r = P_{r_a}$ for ASD. $M_r = M_{r_u}$ for LRFD; $M_r = M_{r_a}$ for ASD.

$$Q_g = \gamma^{0.2} \left[ 1 + \frac{0.024 \gamma^{1.2}}{\exp\left( \frac{0.5g}{t} - 1.33 \right) + 1} \right]$$

Note that $\exp(x)$ is identical to $2.71828^x$, where $2.71828$ is the base of the natural logarithm.

* Equation references are to the AISC Specification.
Table 8-1A. Limits of Applicability of Table 8-1 (also refer to Figure 8-1)

<table>
<thead>
<tr>
<th>Limit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint eccentricity</td>
<td>$-0.55 \leq e/D \leq 0.25$ for K-connections</td>
</tr>
<tr>
<td>Branch angle</td>
<td>$\theta \geq 30^\circ$</td>
</tr>
<tr>
<td>Chord wall slenderness</td>
<td>$D/t \leq 50$ for T-, Y- and K-connections</td>
</tr>
<tr>
<td></td>
<td>$D/t \leq 40$ for cross-connections</td>
</tr>
<tr>
<td>Branch wall slenderness</td>
<td>$D_b/t_b \leq 50$ for tension branch</td>
</tr>
<tr>
<td></td>
<td>$D_b/t_b \leq 0.05E/F_y$ for compression branch</td>
</tr>
<tr>
<td>Width ratio</td>
<td>$0.2 &lt; D_c/D \leq 1.0$ for T-, Y-, cross- and overlapped K-connections</td>
</tr>
<tr>
<td></td>
<td>$0.4 \leq D_c/D \leq 1.0$ for gapped K-connections</td>
</tr>
<tr>
<td>Gap</td>
<td>$g \geq t_{b,\text{comp}} + t_{b,\text{tens}}$ for gapped K-connections</td>
</tr>
<tr>
<td>Overlap</td>
<td>$25% \leq O \leq 100%$ for overlapped K-connections</td>
</tr>
<tr>
<td>Branch thickness</td>
<td>$t_{b,\text{overlapping}} \leq t_{b,\text{overlapped}}$ for branches in overlapped K-connections</td>
</tr>
<tr>
<td>Material strength</td>
<td>$F_y$ and $F_{y,b} \leq 52$ ksi</td>
</tr>
<tr>
<td>Ductility</td>
<td>$F_y/F_s$ and $F_{y,b}/F_{s,b} \leq 0.8$</td>
</tr>
</tbody>
</table>

Note: Limits of applicability are from AISC Specification Section K2.2a.

Table 8-2. Nominal Strengths of Rectangular HSS-to-HSS Truss Connections

<table>
<thead>
<tr>
<th>Connection Type</th>
<th>Connection Nominal Axial Strength*</th>
</tr>
</thead>
</table>
| T-, Y- and Cross-Connections     |                                                                 **Limit State: Chord Wall Plasticity, when $\beta \leq 0.85$**  \[ \begin{align*} P_n \sin \theta &= F_y t^2 \left( \frac{2\beta}{1-\beta} + \frac{4}{\sqrt{1-\beta}} \right) Q_e \tag{K2-13} \end{align*} \]
|                                  | **Limit State: Shear Yielding (Punching), when $0.85 < \beta \leq 1$ or $B/t \leq 10$** \[ \begin{align*} P_n \sin \theta &= 0.6F_y t B \left( 2\beta + 2|t|_{\text{comp}} \right) \tag{K2-14} \end{align*} \]
|                                  | **Limit State: Local Yielding of Chord Sidewalls, when $\beta = 1.0$** \[ \begin{align*} P_n \sin \theta &= 2F_y t(5N + H) \tag{K2-15} \end{align*} \]
|                                  | **Limit State: Local Crippling of Chord Sidewalls, when $\beta = 1.0$ and Branch is in Compression, for T- or Y-Connections** \[ \begin{align*} P_n \sin \theta &= 1.6t^2 \left( 1 + 3N \right) \frac{EF_y}{H - 3t} Q_e \tag{K2-16} \end{align*} \]
|                                  | **Limit State: Local Crippling of Chord Sidewalls, when $\beta = 1.0$ and Branches are in Compression, for Cross-Connections** \[ \begin{align*} P_n \sin \theta &= \left( \frac{48t^3}{H - 3t} \right) \sqrt{EF_y Q_e} \tag{K2-17} \end{align*} \]
|                                  | **Limit State: Local Yielding of Branch/Branches Due to Uneven Load Distribution, when $\beta \geq 0.85$** \[ \begin{align*} P_n &= F_{p,\text{tens}} \left( 2H_b + 2b_{\text{out}} - 4b_b \right) \tag{K2-18} \end{align*} \]
|                                  | **Limit State: Shear of Chord Sidewalls, for Cross-Connections with $\theta < 90^\circ$ and where a Projected Gap Is Created (see figure). Determine $P_n \sin \theta$ in accordance with Spec. Sect. G5.** |
Table 8-2 (continued). Nominal Strengths of Rectangular HSS-to-HSS Truss Connections

<table>
<thead>
<tr>
<th>Connection Type</th>
<th>Connection nominal axial strength*</th>
</tr>
</thead>
</table>
| Gapped K-Connections | Limit State: Chord Wall Plastification, for all $\beta$

\[ P_\alpha \sin \theta = F_t t^2 \left( 9.8 \beta_{\text{eff}} \gamma^{0.5} \right) Q_r \]

\[ \phi = 0.90 \text{ (LRFD)} \quad \Omega = 1.67 \text{ (ASD)} \]

Limit State: Shear Yielding (Punching), when $B_b < B - 2t$

Do not check for square branches.

\[ P_r \sin \theta = 0.6F_t tB(2\eta + \beta + \beta_{\text{eff}}) \]

\[ \phi = 0.95 \text{ (LRFD)} \quad \Omega = 1.58 \text{ (ASD)} \]

Limit State: Shear of Chord Sidewalls in the Gap Region.
Determine $P_r \sin \theta$ in accordance with Spec. Sect. G5.
Do not check for square chords.

Limit State: Local Yielding of Branch/Branches Due to Uneven Load Distribution.
Do not check for square branches or if $B/t \geq 15$

\[ P_h = P_{\text{eff}} t_B \left( 2H_{\text{eff}} + B_b + b_{\text{eff}} - 4t_b \right) \]

\[ \phi = 0.95 \text{ (LRFD)} \quad \Omega = 1.58 \text{ (ASD)} \]

\[ b_{\text{eff}} = \frac{10}{B/t} \left( \frac{F_t}{F_{\text{ref}}} \right) B_b \leq B_b \]

(Note that the force arrows shown for overlapped K-connections may be reversed; $i$ and $j$ control member identification)

Overlapped K-Connections

Limit State: Local Yielding of Branch/Branches Due to Uneven Load Distribution

\[ \phi = 0.95 \text{ (LRFD)} \quad \Omega = 1.58 \text{ (ASD)} \]

For 25% $\leq O_r < 50%$:

\[ P_{h, \text{overlapping branch}} = F_{\text{ref}} t_B \left( \frac{O_r}{50} \left( 2H_{\text{eff}} - 4t_{\text{eff}} \right) + b_{\text{eff}} \right) \]

(K2-24)

For 50% $\leq O_r < 80%$:

\[ P_{h, \text{overlapping branch}} = F_{\text{ref}} t_B \left( 2H_{\text{eff}} - 4t_{\text{eff}} + b_{\text{eff}} \right) \]

(K2-25)

For 80% $\leq O_r < 100%$:

\[ P_{h, \text{overlapping branch}} = F_{\text{ref}} t_B \left( 2H_{\text{eff}} - 4t_{\text{eff}} + B_b + b_{\text{eff}} \right) \]

(K2-26)

where $b_{\text{eff}} = \frac{10}{B/t} \left( \frac{F_t}{F_{\text{ref}}} \right) B_b \leq B_b$

(K2-27)

\[ b_{\text{eff}} = \frac{10}{B_{\text{eff}}/t_{\text{eff}}} \left( \frac{F_{\text{ref}} t_{\text{eff}}}{F_{\text{ref}} t_B} \right) B_b \leq B_{\text{eff}} \]

(K2-28)

Subscript $i$ refers to the overlapping branch
Subscript $j$ refers to the overlapped branch

Subscript $i$ refers to the overlapping branch $P_{\text{h}, \text{overlapping branch}} = \frac{A_i F_{\text{ref}}}{A_{\text{ref}} F_{\text{ref}}}$

Functions

\[ Q_r = 1 \text{ for chord (connecting surface) in tension} \]

\[ Q_r = 1.3 - 0.4 \frac{U}{\beta_{\text{ref}}} \leq 1.0 \text{ for chord (connecting surface) in compression, for T-, Y-, and cross-connections} \]

\[ Q_r = 1.3 - 0.4 \frac{U}{\beta_{\text{ref}}} \leq 1.0 \text{ for chord (connecting surface) in compression, for gapped K-connections} \]

\[ U = \frac{P_r}{AF_c + MS_c} \text{ where } P_r \text{ and } M_r \text{ are determined on the side of the joint that has the higher compression stress. } P_r \text{ and } M_r \text{ refer to required axial and flexural strength in the chord. } P_r = P_r \text{ for LRFD; } P_r = P_r \text{ for ASD. } M_r = M_r \text{ for LRFD; } M_r = M_r \text{ for ASD} \]

\[ \beta_{\text{eff}} = \left( B_b + H_b \right) \text{ compression branch} + \left( B_b + H_b \right) \text{ tension branch} \]

\[ \beta_{\text{eff}} = \frac{5B}{\gamma} \leq \beta \]

* Equation references are to the AISC Specification.
### Table 8-2A. Limits of Applicability of Table 8-2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Limitations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint eccentricity:</td>
<td>$-0.55 \leq e/H \leq 0.25$ for K-connections</td>
</tr>
<tr>
<td>Branch angle:</td>
<td>$\theta \geq 30^\circ$</td>
</tr>
<tr>
<td>Chord wall slenderness:</td>
<td>$B/t$ and $H/t \leq 35$ for gapped K-connections and T-, Y- and cross-connections</td>
</tr>
<tr>
<td>Branch wall slenderness:</td>
<td>$B/t \leq 30$ for overlapped K-connections</td>
</tr>
<tr>
<td></td>
<td>$H/t \leq 35$ for overlapped K-connections</td>
</tr>
<tr>
<td></td>
<td>$B_b/t_b$ and $H_b/t_b \leq 35$ for tension branch</td>
</tr>
<tr>
<td></td>
<td>$\leq 1.25 \left( \frac{E}{F_{\text{eq}}} \right)$ for compression branch of gapped K-, T-, Y- and cross-connections</td>
</tr>
<tr>
<td></td>
<td>$\leq 35$ for compression branch of gapped K-, T-, Y- and cross-connections</td>
</tr>
<tr>
<td></td>
<td>$\leq 1.1 \left( \frac{E}{F_{\text{eq}}} \right)$ for compression branch of overlapped K-connections</td>
</tr>
<tr>
<td>Width ratio:</td>
<td>$B_b/B$ and $H_b/B \geq 0.25$ for T-, Y-, cross- and overlapped K-connections</td>
</tr>
<tr>
<td>Aspect ratio:</td>
<td>$0.5 \leq H_b/B_b \leq 2.0$ and $0.5 \leq H/B \leq 2.0$</td>
</tr>
<tr>
<td>Overlap:</td>
<td>$25% \leq O_b \leq 100%$ for overlapped K-connections</td>
</tr>
<tr>
<td>Branch width ratio:</td>
<td>$B_b/B_{ij} \geq 0.75$ for overlapped K-connections, where subscript $i$ refers to the overlapping branch and subscript $j$ refers to the overlapped branch</td>
</tr>
<tr>
<td>Branch thickness ratio:</td>
<td>$t_b/t_{ij} \leq 1.0$ for overlapped K-connections, where subscript $i$ refers to the overlapping branch and subscript $j$ refers to the overlapped branch</td>
</tr>
<tr>
<td>Material strength:</td>
<td>$F_y$ and $F_{\text{eq}} \leq 52$ ksi (360 MPa)</td>
</tr>
<tr>
<td>Ductility:</td>
<td>$F_y/F_u$ and $F_{\text{eq}}/F_{\text{eq,ul}} \leq 0.8$</td>
</tr>
</tbody>
</table>

**Note:** Limits of applicability are from AISC Specification Section K2.3a.

### Additional Limits for Gapped K-Connections

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Limitations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width ratio:</td>
<td>$B_b/B$ and $H_b/B \geq 0.1 + \frac{\gamma}{50}$</td>
</tr>
<tr>
<td></td>
<td>$\beta_{\text{eff}} \geq 0.35$</td>
</tr>
<tr>
<td>Gap ratio:</td>
<td>$\zeta = g/B \geq 0.5(1 - \beta_{\text{eff}})$ Spec. Sect. K2.3c</td>
</tr>
<tr>
<td>Gap:</td>
<td>$g \geq t_b$ compression branch, $+t_b$ tension branch</td>
</tr>
<tr>
<td>Branch size:</td>
<td>smaller $B_b &gt; 0.63$ (larger $B_b$), if both branches are square</td>
</tr>
</tbody>
</table>

**Note:** Maximum gap size will be controlled by the $e/H$ limit. If gap is large, treat as two Y-connections.
8.6 CONNECTION DESIGN EXAMPLES

Example 8.1—Y-Connection with Round HSS

Given:

Determine the adequacy of the welded HSS Y-connection under the member loads shown in Figure 8-6. Loads consist of 25% dead load and 75% live load. Assume the welds are strong enough to develop the yield strength of the connected branch wall at all locations around the branch.

From AISC Manual Table 2-3, the material properties are as follows:

- Both members
  - ASTM A500 Grade B
  - $F_y = 42$ ksi
  - $F_u = 58$ ksi

From AISC Manual Table 1-13, the HSS geometric properties are as follows:

- HSS 6.000×0.375
  - $D = 6.00$ in.
  - $t = 0.349$ in.
  - $A = 6.20$ in.$^2$

- HSS 4.000×0.250
  - $D_b = 4.00$ in.
  - $t_b = 0.233$ in.
  - $A_b = 2.76$ in.$^2$

Solution:

Limits of applicability

From AISC Specification Section K2.2a and Table 8-1A, the limits of applicability for this connection are:

- $\theta = 45^\circ \geq 30^\circ$ o.k.
- $D/t \leq 17.2 \leq 50$ o.k.
- $D_b/t_b \leq 17.2 \leq 50$ o.k.
- $0.2 < D_b/D = 0.667 \leq 1.0$ o.k.

- $F_y = F_{y,b}$
  - $F_y = 42$ ksi ≤ 52 ksi o.k.
- $F_{y,b}/F_u = F_{y,b}/F_{u,b}$
  - $F_{y,b}/F_u = 0.724 \leq 0.8$ o.k.
**Required strength (expressed as a force in the branch)**

From Chapter 2 of ASCE 7, the required strength of the connection based on the applied axial loads in the branch is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \( P_u = 1.2(15.0 \text{ kips}) + 1.6(45.0 \text{ kips}) \) | \( P_u = 15.0 \text{ kips} + 45.0 \text{ kips} \)  
\( = 90.0 \text{ kips} \) | \( = 60.0 \text{ kips} \) |

**Chord plastification**

From AISC Specification Section K2.2b(a), the nominal strength for the limit state of chord plastification of the branch is,

\[
P_{\text{sin}\theta} = F_c \sqrt{3.1 + 15.6\beta^2} \gamma^2 Q_f
\]

(Spec. Eq. K2-3 and Table 8-1)

where

\( \beta = D_c/D \)
\( = 0.667 \)
\( \gamma = D/2t \)
\( = 8.60 \)
\( Q_f = 1.0 - 0.3U (1 + U) \)
(Spec. Eq. K2-1 and Table 8-1)

\[
U = \frac{P_{\text{AF}} + M_r}{AF_c + SF_c}
\]
for chord in compression
(Spec. Eq. K2-2 and Table 8-1)

From Chapter 2 of ASCE 7, the required strength in the chord on the side of the joint with the lower compression stress is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \( P_r = P_u \) | \( P_r = P_u \)  
\( = 1.2(10.0 \text{ kips}) + 1.6(30.0 \text{ kips}) \) | \( = 10.0 \text{ kips} + 30.0 \text{ kips} \)  
\( = 60.0 \text{ kips in. chord on side of joint with lower compression stress} \) | \( = 40.0 \text{ kips in. chord on side of joint with lower compression stress} \)  
| \( M_r = 0 \) | \( M_r = 0 \)  
\( F_c = F_c \) | \( F_c = 0.6F_c \)  
\( = 42 \text{ ksi} \) | \( = 25.2 \text{ ksi} \) |
\( U = \frac{60.0 \text{ kips}}{6.20 \text{ in.}^2 (42 \text{ ksi})} \) | \( U = \frac{40.0 \text{ kips}}{6.20 \text{ in.}^2 (25.2 \text{ ksi})} \)  
\( = 0.230 \) | \( = 0.256 \) |
\( Q_f = 1.0 - 0.3(0.230)(1 + 0.230) \) | \( Q_f = 1.0 - 0.3(0.256)(1 + 0.256) \)  
\( = 0.915 \) | \( = 0.904 \) |
The available strength for the limit state of chord plastification is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_n = 42 \text{ ksi} \left( 0.349 \text{ in.} \right)^2 \left[ 3.1 + 15.6 \left( 0.667 \right)^2 \right] \times \left( 8.60 \right)^{0.2} \left( 0.915 \right) / \sin 45^\circ )</td>
<td>( P_n = 42 \text{ ksi} \left( 0.349 \text{ in.} \right)^2 \left[ 3.1 + 15.6 \left( 0.667 \right)^2 \right] \times \left( 8.60 \right)^{0.2} \left( 0.904 \right) / \sin 45^\circ )</td>
</tr>
<tr>
<td>= 102 kips</td>
<td>= 101 kips</td>
</tr>
<tr>
<td>( \phi P_n = 0.90 \left( 102 \text{ kips} \right) )</td>
<td>( \phi P_n = 0.90 \left( 101 \text{ kips} \right) )</td>
</tr>
<tr>
<td>= 91.8 kips</td>
<td>( \phi P_n = 0.90 \left( 101 \text{ kips} \right) )</td>
</tr>
<tr>
<td>91.8 kips &gt; 90.0 kips o.k.</td>
<td>60.5 kips &gt; 60.0 kips o.k.</td>
</tr>
</tbody>
</table>

**Shear yielding (punching)**

According to AISC Specification Section K2.2b(b), the limit state of shear yielding (punching) need not be checked when \( \beta > (1 - 1/y) \). Because \( \beta \leq (1 - 1/y) \), which reduces to \( D_b = 4.00 \text{ in.} \). \( (D-2t) = 5.30 \text{ in.} \), this limit state must be considered. From AISC Specification Section K2.2b(b), the nominal strength is:

\[
P_n = 0.6 F_{y} \pi D_b \left[ \frac{1 + \sin \theta}{2 \sin^2 \theta} \right] = 0.6 \left( 42 \text{ ksi} \right) \left( 0.349 \text{ in.} \right) \pi \left( 4.00 \text{ in.} \right) \left[ \frac{1 + \sin 45^\circ}{2 \sin^2 45^\circ} \right]
\]

\( = 189 \text{ kips} \)

The available strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi P_n = 0.95 \left( 189 \text{ kips} \right) )</td>
<td>( \phi P_n = 0.95 \left( 189 \text{ kips} \right) )</td>
</tr>
<tr>
<td>= 180 kips</td>
<td>= 180 kips</td>
</tr>
<tr>
<td>180 kips &gt; 90.0 kips o.k.</td>
<td>120 kips &gt; 60.0 kips o.k.</td>
</tr>
</tbody>
</table>

**Example 8.2—Overlapped K-Connection with Round HSS**

**Given:**

Verify the adequacy of the welded N-connection (a particular case of K-connection) shown in the free-body diagram in Figure 8-7. Note that the chord moment is necessary for equilibrium because of the noding eccentricity. The connection is a balanced K-connection because the load in the vertical compression branch member is equilibrated (within 20%) by the vertical component of the tension branch member [see AISC Specification Section K2(b)]. Note that the thicker branch member is the through branch. For fabrication, the compression (through) branch is tacked initially to the chord, the diagonal (overlapping) branch is then tacked into place, and finally the whole connection is welded together. The loads shown consist of live and dead loads in the ratio of 2.5:1. Assume the welds are strong enough to develop the yield strength of the connected branch wall at all locations around the branch.

From AISC Manual Table 2-3, the material properties are as follows:

- All members
  - ASTM A500 Grade C
  - \( F_y = 46 \text{ ksi} \)
  - \( F_u = 62 \text{ ksi} \)
From AISC Manual Table 1-13, the HSS geometric properties are as follows:

HSS10.000×0.500
- \( A = 13.9 \text{ in}^2 \)
- \( D = 10.000 \text{ in.} \)
- \( t = 0.465 \text{ in.} \)

HSS6.625×0.375
- \( A_b = 6.88 \text{ in}^2 \)
- \( D_b = 6.625 \text{ in.} \)
- \( t_b = 0.349 \text{ in.} \)

HSS6.625×0.250
- \( A_b = 4.68 \text{ in}^2 \)
- \( D_b = 6.625 \text{ in.} \)
- \( t_b = 0.233 \text{ in.} \)

Solution:

Limits of applicability

The connection noding eccentricity, \( e = -2.50 \text{ in.} \) (negative because the branch centerlines intersect toward the inside of the truss, relative to the chord centerline).

\[ q = \text{overlap length measured along the connecting face of the chord beneath the two branches} \]

\[
q = \frac{D_b + D_{b2}}{2\sin\theta_{b1}} - \left( \frac{e + D/2}{\sin\theta_{b1} \sin\theta_{b2}} \right) - \left( \frac{2.50 \text{ in.} + 5.00 \text{ in.}}{\sin 90^\circ \sin 50^\circ} \right) - \left( \frac{2.50 \text{ in.}}{\sin 140^\circ} \right)
\]

\[ = 7.64 \text{ in.} - 2.10 \text{ in.} = 5.54 \text{ in.} \]

\( P_i = 57.1 \text{ kips} \)
\( P_D = 22.9 \text{ kips} \)

\( P_i = 74.3 \text{ kips} \)
\( P_D = 29.7 \text{ kips} \)

\( P_L = 82.9 \text{ kips} \)
\( P_D = 33.1 \text{ kips} \)

\( M_L = 119 \text{ kip-in.} \)
\( M_D = 47.7 \text{ kip-in.} \)

Fig. 8-7. Welded overlap K-connection with round HSS.
\[ p = \text{projected length of the overlapping branch on the chord} \]
\[ = 6.25 \text{ in.} / \sin 50^\circ \]
\[ = 8.65 \text{ in.} \]
\[ O_e = \left( \frac{q}{p} \right) (100\%) \]
\[ = \frac{5.54}{8.65} \text{ in.} (100\%) \]
\[ = 64.0\% \]
\[ -0.55 \leq e/D = -0.250 \leq 0.25 \quad \text{o.k.} \]

As the noding eccentricity satisfies this limit, the resulting total eccentricity moment that it produces \([(2.5 \text{ in.})(104 \text{ kips}) \times \cos 50^\circ) = 167 \text{ kip-in.}] can be neglected with regard to connection design. (However, it would still have an effect on chord member design, in general.) Figure 8-7 shows the eccentricity moment to the right of the connection, but in general it would be distributed to both the left and right of the connection in proportion to the chord stiffness. The branches attract very little moment.

\[ \theta_{st} = 90^\circ \geq 30^\circ; \theta_{st} = 50^\circ \geq 30^\circ \quad \text{o.k.} \]
\[ D/t = 21.5 \leq 50 \quad \text{o.k.} \]
\[ D_e/t_s = 28.4 \leq 50 \quad \text{for tension branch} \quad \text{o.k.} \]
\[ D_e/t_s = 19.0 \leq 0.05E/F_{yd} = 31.5 \quad \text{for compression branch} \quad \text{o.k.} \]
\[ 0.2 < D_e/D = 0.663 \leq 1.0 \quad \text{o.k.} \]
\[ 25\% \leq O_e = 64.0\% \leq 100\% \quad \text{o.k.} \]
\[ t_{st} = 0.233 \text{ in.} \leq t_{sy} = 0.349 \text{ in.} \quad \text{o.k.} \]
\[ F_{y} = F_{yd} \]
\[ = 46 \text{ ksi} \leq 52 \text{ ksi} \quad \text{o.k.} \]
\[ F_{y}/F_{yd} = F_{yd}/F_{yd} \]
\[ = 0.742 \leq 0.8 \quad \text{o.k.} \]

**Required strength (expressed as a force in a branch)**

From Chapter 2 of ASCE 7, the required strengths are:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>For compression branch,</td>
<td>( P_u = 1.2 \left( 22.9 \text{ kips} \right) + 1.6 \left( 57.1 \text{ kips} \right) ) = 119 kips</td>
<td>( P_u = 22.9 \text{ kips} + 57.1 \text{ kips} ) = 80.0 kips</td>
</tr>
<tr>
<td>For tension branch,</td>
<td>( P_u = 1.2 \left( 29.7 \text{ kips} \right) + 1.6 \left( 74.3 \text{ kips} \right) ) = 155 kips</td>
<td>( P_u = 29.7 \text{ kips} + 74.3 \text{ kips} ) = 104 kips</td>
</tr>
</tbody>
</table>

**Limit state of chord plastification**

From AISC Specification Section K2.2c, the nominal strength for the limit state of chord plastification is:

\[
\left( P_u \sin \theta \right)_{\text{compression branch}} = F_{yt}^2 \left( 2.0 + 11.33 \frac{D_e}{D \text{,req}} \right) Q_i Q_f
\]

\((Spec. \ Eq. K2-6 \text{ and Table 8-1})\)
where

\[ Q_f = 1.0 \text{ as the chord is in tension} \]

\[ Q_s = \gamma \left[ 1 + \frac{0.024 \gamma^2}{\exp \left( \frac{0.5 g}{t} - 1.33 \right) + 1} \right]^2 \]

\[ g = -q = -5.54 \text{ in.} \]
\[ \gamma = \frac{D}{2t} = 10.8 \]

Therefore

\[ Q_s = 10.8^{0.2} \left[ 1 + \frac{0.024 (10.8)^2}{\exp \left( -0.5 \frac{5.54 \text{ in.}}{0.465 \text{ in.}} - 1.33 \right) + 1} \right] \]
\[ = 2.28 \]

and

\[ (P_n \sin \theta)_{\text{compression branch}} = 46 \text{ ksi } (0.465 \text{ in.})^2 \left[ 2.0 + 11.33 \frac{6.625 \text{ in.}}{10.0 \text{ in.}} \right] (2.28)(1.0) \]
\[ = 216 \text{ kips} \]

\[ P_n \text{ compression branch} = 216 \text{ kips} \sin 90^\circ \]
\[ = 216 \text{ kips} \]

\[ (P_n \sin \theta)_{\text{tension branch}} = (P_n \sin \theta)_{\text{compression branch}} \]
\[ = 216 \text{ kips} \]

\[ P_n \text{ tension branch} = 216 \text{ kips} \sin 50^\circ \]
\[ = 282 \text{ kips} \]

The available strengths for chord plastification expressed as forces in the tension and compression branches are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| For compression branch, \[ \phi P_n = 0.90(216 \text{ kips}) \] = 194 kips \[ 194 \text{ kips} > 119 \text{ kips} \text{ o.k.} \] | For compression branch, \[ P_n = 216 \text{ kips} \]
\[ \frac{P_n}{\Omega} = \frac{216}{1.67} = 129 \text{ kips} \]
\[ 129 \text{ kips} > 80.0 \text{ kips} \text{ o.k.} \] | For tension branch, \[ P_n = 282 \text{ kips} \]
\[ \frac{P_n}{\Omega} = \frac{282}{1.67} = 169 \text{ kips} \]
\[ 169 \text{ kips} > 104 \text{ kips} \text{ o.k.} \] |
Example 8.3—Cross-Connection with Rectangular HSS

Given:

Determine the adequacy of the welded HSS connection illustrated in Figure 8-8 subject to the loads indicated. The connection behaves as a planar cross-connection because the punching load is transmitted through the chord member and is equilibrated by branch member forces on the opposite side of the chord. The two branches on one side of the chord, having a gap at the branch toes of 0.960 in. (just sufficient for welding), can be considered to have a composite effect on the chord. Assume the welds are strong enough to develop the yield strength of the connected branch wall at all locations around the branch.

From AISC Manual Table 2-3, the material properties are as follows:

- All members
- ASTM A500 Grade B
- $F_y = 46$ ksi
- $F_u = 58$ ksi

From AISC Manual Tables 1-11 and 1-12, the HSS geometric properties are as follows:

HSS $8 \times 8 \times \frac{3}{4}$
- $A = 7.10$ in.$^2$
- $B = 8.00$ in.
- $H = 8.00$ in.
- $t = 0.233$ in.

HSS $8 \times 6 \times \frac{3}{8}$
- $A_b = 8.97$ in.
- $B_b = 8.00$ in.
- $H_b = 6.00$ in.
- $t_b = 0.349$ in.

Fig. 8-8. Welded cross-connection with rectangular HSS.
Solution:

**Limits of applicability**

From AISC Specification Section K2.3a and Table 8-2A, the limits of applicability for rectangular HSS are:

\[
\begin{align*}
\theta &\quad = 35^\circ \geq 30^\circ \quad \text{o.k.} \\
\frac{B}{t} &\quad = \frac{H}{t} \\
&\quad = 34.3 \leq 35 \quad \text{o.k.} \\
\frac{H_b}{t_b} &\quad = \frac{6.00 \text{ in.}}{0.349 \text{ in.}} \\
&\quad = 17.2 \leq 1.25 \sqrt[2]{\frac{E}{F_{sb}}} \\
&\quad = 1.25 \sqrt[2]{\frac{E}{F_{sb}}} = 1.25 \sqrt[2]{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} \\
&\quad = 31.4 \\
17.2 &\quad \leq 31.4 \quad \text{o.k.} \\
\frac{B_b}{t_b} &\quad = \frac{8.00 \text{ in.}}{0.349 \text{ in.}} \\
&\quad = 22.9 \leq 1.25 \sqrt[2]{\frac{E}{F_{sb}}} \\
22.9 &\quad \leq 31.4 \quad \text{o.k.} \\
\frac{H_b}{t_b} &\quad = 17.2 \leq 35 \quad \text{o.k.} \\
\frac{B_b}{t_b} &\quad = 22.9 \leq 35 \quad \text{o.k.} \\
\frac{B_b}{B} &\quad = \frac{8.00 \text{ in.}}{8.00 \text{ in.}} \\
&\quad = 1.00 \geq 0.25 \quad \text{o.k.} \\
\frac{H_b}{B} &\quad = \frac{6.00 \text{ in.}}{8.00 \text{ in.}} \\
&\quad = 0.750 \geq 0.25 \quad \text{o.k.} \\
0.5 \leq \frac{H_b}{B} &\quad \leq 2.0 \\
\frac{H_b}{B_b} &\quad = \frac{6.00 \text{ in.}}{8.00 \text{ in.}} \\
&\quad = 0.750 \\
0.5 \leq 0.750 &\quad \leq 2.0 \quad \text{o.k.}
\end{align*}
\]
\[ 0.5 \leq \frac{H}{B} \leq 2.0 \]
\[ \frac{H}{B} = \frac{8.00 \text{ in.}}{8.00 \text{ in.}} = 1.00 \]

\[ 0.5 \leq 1.00 \leq 2.0 \quad \text{o.k.} \]

\[ F_y = F_{yb} \]
\[ = 46 \text{ ksi} \leq 52 \text{ ksi} \quad \text{o.k.} \]

\[ \frac{F_y}{F_{yb}} = \frac{F_y}{F_{yb}} \]
\[ = \frac{46 \text{ ksi}}{58 \text{ ksi}} \]
\[ = 0.793 \leq 0.8 \quad \text{o.k.} \]

*Required strength (expressed as a force in a branch)*

From Chapter 2 of ASCE 7, the required strength of the connection expressed as a force in the branch is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[ P_u = 1.2(25.0 \text{ kips}) + 1.6(25.0 \text{ kips}) \]  
\[ = 70.0 \text{ kips} \] | \[ P_u = 25.0 \text{ kips} + 25.0 \text{ kips} \]  
\[ = 50.0 \text{ kips} \] |

*Local yielding of chord sidewalls*

Because \( \beta = B_s / B = 1.00 \), the limit state of local yielding of the chord sidewalls is applicable. From AISC Specification Section K2.3b(c), the nominal strength is:

\[ P_{n \sin \theta} = 2F_y t \left(5k + N\right) \]  
\[ (\text{Spec. Eq. K2-15 and Table 8-2}) \]

where

\[ k = \text{outside corner radius of HSS} \]
\[ = 1.5t \]
\[ = 0.350 \text{ in.} \]

\[ N = \text{bearing length on chord} \]
\[ = 2 \left(6.00 \text{ in.} / \sin 35^\circ\right) + 0.960 \text{ in.} \]
\[ = 21.9 \text{ in.} \]

Therefore, the nominal strength is:

\[ P_{n \sin \theta} = 2(46 \text{ ksi})(0.233 \text{ in.}) \left[5(0.350 \text{ in.}) + 21.9 \text{ in.}\right] \]
\[ = 507 \text{ kips} \]

\[ P_n = \frac{507}{\left(2\right) \sin 35^\circ} \]
\[ = 442 \text{ kips per branch} \]
The available strength for the limit state of local yielding of the chord sidewalls is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi P_n = 1.00 \left(442 \text{ kips} \right) )</td>
<td>( P_n = 442 \text{ kips} )</td>
</tr>
<tr>
<td>= 442 kips</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>442 kips &gt; 70.0 kips ( \text{o.k.} )</td>
<td>= 295 kips</td>
</tr>
<tr>
<td></td>
<td>295 kips &gt; 50.0 kips ( \text{o.k.} )</td>
</tr>
</tbody>
</table>

**Local crippling of the chord sidewalls**

From AISC Specification Section K2.3b(c), the nominal strength for the limit state of local crippling of the chord sidewalls is:

\[
P_n \sin \theta = \left( \frac{48 t^3}{H - 3t} \right) \sqrt{0.8 \cdot 0.233 \text{ in.}^3} \cdot \sqrt{29,000 \text{ ksi} \cdot 46 \text{ ksi} \cdot 1.0}
\]

where

\( Q_f = 1.0 \) for chord in tension in accordance with AISC Specification Section K2.3

Therefore, the nominal strength is:

\[
P_n \sin \theta = \left[ \frac{48 \left(0.233 \text{ in.}\right)^3}{8.00 \text{ in.} - 3(0.233 \text{ in.})} \right] \sqrt{29,000 \text{ksi} \cdot 46 \text{ksi} \cdot 1.0}
\]

\( = 96.1 \text{kips} \)

\( P_n = \left(96.1 / \sin 35^\circ\right)/2 = 83.8 \text{ kips per branch} \)

The available strength for the limit state of local crippling of the chord sidewalls is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi P_n = 0.90 \left(83.8 \text{ kips} \right) )</td>
<td>( P_n = 83.8 \text{ kips} )</td>
</tr>
<tr>
<td>= 75.4 kips</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>75.4 kips &gt; 70.0 kips ( \text{o.k.} )</td>
<td>= 50.2 kips</td>
</tr>
<tr>
<td></td>
<td>50.2 kips &gt; 50.0 kips ( \text{o.k.} )</td>
</tr>
</tbody>
</table>

**Local yielding of branches due to uneven load distribution**

From AISC Specification Section K2.3b(d), the available strength for the limit state of local yielding due to uneven load distribution is determined as follows:

\[
P_n = F_{a,t_b} \left(2 H_b + 2b_{su} - 4t_b\right)
\]

\( (\text{Spec. Eq. K2-18 and Table 8-2}) \)
where 

\[ b_{	ext{eff}} = \text{effective width of branch face transverse to the chord; it will be conservative to assume this reduced effective width for all branch walls } B_b \]

\[
= \frac{10}{B/t} \left( \frac{F_{y,t}}{F_{y,t}} \right) \left( \frac{F_{y,t}}{F_{y,t}} \right) B_b \\
= 10 \left[ \frac{46 \text{ ksi (0.233 in.)}}{34.3 \text{ ksi (0.349 in.)}} \right] (8.00 \text{ in.}) \\
= 1.56 \text{ in.}
\]

Therefore

\[ P_n = 46 \text{ ksi (0.349 in.)} \left[ 2(6.00 \text{ in.}) + 2(1.56 \text{ in.}) - 4(0.349 \text{ in.}) \right] \]

\[ = 220 \text{ kips per branch} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi P_n = 0.95(220 \text{ kips}) )</td>
<td>( P_n = 220 \text{ kips} )</td>
</tr>
<tr>
<td>= 209 kips</td>
<td>( \Omega = 1.58 )</td>
</tr>
<tr>
<td>209 kips &gt; 70.0 kips <strong>O.K.</strong></td>
<td>( = 139 \text{ kips} )</td>
</tr>
<tr>
<td>139 kips &gt; 50.0 kips <strong>O.K.</strong></td>
<td>( = 139 \text{ kips} )</td>
</tr>
</tbody>
</table>

Although there is a vertical plane through the center of the connection that could serve as a shear plane, the limit state of shear on the chord sidewalls is not checked because there is no net shearing force on the other side of this plane.

Other limit states for cross-connections (see AISC Specification Section K2.3b and Table 8-2) do not control because \( \beta = B_b/B = 1.00 \).

**Example 8.4—Overlapped K-Connection with Rectangular HSS**

**Given:**

A planar roof truss contains the welded HSS 45° overlapped K-connection shown in Figure 8-9. Note that the chord moment is necessary for equilibrium because of the noding eccentricity. The connection is a balanced K-connection because the vertical component of the compression branch member force is equilibrated (within 20%) by the vertical component of the tension branch member force [see AISC Specification Section K2(b)]. The through branch is the wider and thicker branch member. For fabrication the compression (through) branch is tacked initially to the chord, the diagonal (overlapping) branch is then tacked into place, and finally the whole connection is welded together. The loads shown consist of live load and dead load in the ratio of 3:1. Determine the adequacy of the connection under the given loads. Assume the welds are strong enough to develop the yield strength of the connected branch wall at all locations around the branch.

![Fig. 8-9. Welded overlap K-connection with rectangular HSS.](image-url)
From AISC Manual Table 2-3, the HSS material properties are as follows:

All members
ASTM A500 Grade B
\( F_y = F_{yb} \)
\( = 46 \text{ ksi} \)
\( F_u = F_{ub} \)
\( = 58 \text{ ksi} \)

From AISC Manual Tables 1-11 and 1-12, the HSS geometric properties are as follows:

<table>
<thead>
<tr>
<th>HSS</th>
<th>( A )</th>
<th>( B )</th>
<th>( H )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8\times8\times1/2</td>
<td>13.5 in.²</td>
<td>8.00 in.</td>
<td>8.00 in.</td>
<td>0.465 in.</td>
</tr>
<tr>
<td>4\times4\times1/16</td>
<td>5.26 in.²</td>
<td>4.00 in.</td>
<td>6.00 in.</td>
<td>0.291 in.</td>
</tr>
<tr>
<td>3\times3\times1/8</td>
<td>3.37 in.²</td>
<td>3.00 in.</td>
<td>5.00 in.</td>
<td>0.233 in.</td>
</tr>
</tbody>
</table>

Solution:

Limits of applicability

Check the limits of applicability for rectangular HSS given in AISC Specification Section K2.3a and Table 8-2A.

Connection nodding eccentricity, \( e = -2.00 \text{ in.} \) (negative because the branch centerlines intersect toward the branches, relative to the chord centerline).

\( q = \text{overlap length measured along the connecting face of the chord beneath the two branches, from geometry} \)

\[
q = \left( \frac{H_{bj}}{2\sin\theta_{bj}} + \frac{H_{bi}}{2\sin\theta_{bi}} \right) - \left( \frac{e + H/2}{\sin\theta_{bj} \sin\theta_{bi}} \right) \frac{\sin(\theta_{bj} + \theta_{bi})}{\sin(\theta_{bj} + \theta_{bi})} \]

\[
= \left( \frac{6.00 \text{ in.}}{2\sin45^\circ} + \frac{5.00 \text{ in.}}{2\sin45^\circ} \right) - \left( \frac{-2.00 \text{ in.} + 4.00 \text{ in.}}{(\sin45^\circ)^2} \right) \frac{\sin90^\circ}{\sin90^\circ} \]

\( = 7.78 \text{ in.} - 4.00 \text{ in.} \)
\( = 3.78 \text{ in.} \)

\( p = \text{projected length of the overlapping branch on the chord} \)

\( = \frac{5.00 \text{ in.}}{\sin45^\circ} \)
\( = 7.07 \text{ in.} \)
\[ O_e = \left( \frac{q}{p} \right) (100\%) \]
\[ = \frac{3.78 \text{ in.}}{7.07 \text{ in.}} (100\%) \]
\[ = 53.5\% \]
\[ -0.55 \leq \frac{e}{H} = -0.250 \leq 0.25 \quad \text{o.k.} \]

As the noding eccentricity satisfied this limit, the resulting total eccentricity moment that it produces \([2(2.00 \text{ in.})(100 \text{ kips}) (\cos 45^\circ) = 283 \text{ kip-in.}]\) can be neglected with regard to connection design (however, it would still have an effect on chord member design in general).

\[ \theta_{h_i} = \theta_{b_j} \]
\[ = 45^\circ \geq 30^\circ \quad \text{o.k.} \]

\[ B \]
\[ t \]
\[ = \frac{8.00 \text{ in.}}{0.465} \]
\[ = 17.2 \leq 30 \quad \text{o.k.} \]

\[ H \]
\[ t \]
\[ = \frac{8.00 \text{ in.}}{0.465 \text{ in.}} \]
\[ = 17.2 \leq 35 \quad \text{o.k.} \]

For the tension branch

\[ B_{h_i} = \frac{3.00 \text{ in.}}{t_{h_i} = 0.233 \text{ in.}} \]
\[ = 12.9 \leq 35 \quad \text{o.k.} \]

\[ H_{h_i} = \frac{21.5 \leq 35 \quad \text{o.k.}}{t_{h_i} =} \]

For the compression branch

\[ B_{b_j} = \frac{4.00 \text{ in.}}{t_{b_j} = 0.291 \text{ in.}} \]
\[ = 13.7 \leq 1.1 \sqrt[3]{\frac{E}{F_{y_{b_j}}}} \]
\[ = 13.7 \leq 27.6 \quad \text{o.k.} \]

\[ H_{b_j} = \frac{6.00 \text{ in.}}{t_{b_j} = 0.291} \]
\[ = 20.6 \leq 1.1 \sqrt[3]{\frac{E}{F_{y_{b_j}}}} \]
\[ = 20.6 \leq 27.6 \quad \text{o.k.} \]
For the tension branch
\[ \frac{B_{ti}}{B} = \frac{3.00 \text{ in.}}{8.00 \text{ in.}} = 0.375 \geq 0.25 \quad \text{o.k.} \]
\[ \frac{H_{ti}}{B} = \frac{5.00 \text{ in.}}{8.00 \text{ in.}} = 0.625 \geq 0.25 \quad \text{o.k.} \]

For the compression branch
\[ \frac{B_{cj}}{B} = \frac{4.00 \text{ in.}}{8.00 \text{ in.}} = 0.500 \geq 0.25 \quad \text{o.k.} \]
\[ \frac{H_{cj}}{B} = \frac{6.00 \text{ in.}}{8.00 \text{ in.}} = 0.750 \geq 0.25 \quad \text{o.k.} \]

For the tension branch
\[ 0.5 \leq \frac{H_{ti}}{B_{ti}} \leq 2.0 \]
\[ \frac{H_{ti}}{B_{ti}} = \frac{5.00 \text{ in.}}{3.00 \text{ in.}} = 1.67 \]
\[ 0.5 \leq 1.67 \leq 2.0 \quad \text{o.k.} \]

For the compression branch
\[ 0.5 \leq \frac{H_{cj}}{B_{cj}} \leq 2.0 \]
\[ \frac{H_{cj}}{B_{cj}} = \frac{6.00 \text{ in.}}{4.00 \text{ in.}} = 1.50 \]
\[ 0.5 \leq 1.50 \leq 2.0 \quad \text{o.k.} \]

For the chord
\[ 0.5 \leq \frac{H}{B} = \frac{1.00}{2.00} = 0.50 \quad \text{o.k.} \]
\[ 25\% \leq O_v = 5.5\% \leq 100\% \quad \text{o.k.} \]

The width of the overlapping branch, \( B_{ti} \), divided by the width of the overlapped branch, \( B_{cj} \), must be greater than or equal to 0.75, where \( B_{ti} \) and \( B_{cj} \) are the branch widths perpendicular to the longitudinal axis of the chord.
\[ \frac{B_{ti}}{B_{cj}} = 0.750 \geq 0.75 \quad \text{o.k.} \]
\[ \frac{t_{ti}}{t_{cj}} = 0.801 \leq 1.0 \quad \text{o.k.} \]
\[ F_y = F_{yb} \]
\[ = 46 \text{ ksi} \leq 52 \text{ ksi} \quad \text{o.k.} \]

\[ F_y = F_{ub} \]
\[ = 46 \text{ ksi} \]
\[ = 58 \text{ ksi} \]
\[ = 0.793 \leq 0.8 \quad \text{o.k.} \]

**Required strength (expressed as a force in a branch)**

From Chapter 2 of ASCE 7, the required strength of the connection, expressed as a force in the tension and compression branches is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>For compression</td>
<td>( F_{ub} ) for tension branch, ( P_u = 1.2(25.0 \text{ kips}) + 1.6(75.0 \text{ kips}) )</td>
<td>For compression branch and tension branch, ( P_u = 25.0 \text{ kips} + 75.0 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>( 150 \text{ kips} )</td>
<td>( 100 \text{ kips} )</td>
</tr>
</tbody>
</table>

**Local yielding of branches due to uneven load distribution**

From AISC Specification Section K2.3d, the nominal strength of the overlapping branch for the limit state of local yielding due to uneven load distribution is:

\[ P_{n, \text{ overlapping branch}} = F_{iyb} t_{iy} \left( 2H_{bi} - 4t_{bi} + b_{\text{ovl}} + b_{\text{ovv}} \right) \quad \text{for} \ 50\% \leq \theta_y < 80\% \]  
(Spec. Eq. K2-25 and Table 8-2)

where

\[ b_{\text{ovl}} = \frac{10}{B_i / t_i} \left( \frac{F_y t}{F_{iyb} t_{iy}} \right) B_{bi} \leq B_{bi} \]  
(Spec. Eq. K2-27 and Table 8-2)

\[ = \frac{10}{17.2} \left[ 46 \text{ ksi} \left( 0.465 \text{ in.} \right) \right] \left( 3.00 \text{ in.} \right) \]
\[ = 3.48 \text{ in.} > B_{bi} = 3.00 \text{ in.}, \text{ therefore use } b_{\text{ovl}} = 3.00 \text{ in.} \]

\[ b_{\text{ovv}} = \frac{10}{B_{bi} / t_{bi}} \left( \frac{F_{iyb} t_{iy}}{F_{iyb} t_{iy}} \right) B_{bi} \leq B_{bi} \]  
(Spec. Eq. K2-28 and Table 8-2)

\[ = \frac{10}{13.7} \left[ 46 \text{ ksi} \left( 0.291 \text{ in.} \right) \right] \left( 3.00 \text{ in.} \right) \]
\[ = 2.73 \text{ in.} \leq B_{bi} = 3.00 \text{ in.}, \text{ therefore } b_{\text{ovv}} = 2.73 \text{ in.} \]

Therefore, the nominal strength of the overlapping branch is:

\[ P_{n, \text{ overlapping branch}} = 46 \text{ ksi} \left[ 2(5.00 \text{ in}) - 4(0.233 \text{ in.}) + 3.00 \text{ in.} + 2.73 \text{ in.} \right] \]
\[ = 159 \text{ kips} \]

The nominal strength of the overlapped branch is:

\[ P_{n, \text{ overlapped branch}} = P_{n, \text{ overlapping branch}} \left( \frac{A_{iy} F_{iyb}}{A_{iy} F_{iyb}} \right) \]
\[ = 159 \text{ kips} \left[ \frac{5.26 \text{ in.}^2 (46 \text{ ksi})}{3.37 \text{ in.}^2 (46 \text{ ksi})} \right] \]
\[ = 248 \text{ kips} \]
The available connection strength, expressed as forces in the tension (overlapping) and compression (overlapped) branches, is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>For tension (overlapping) branch, ( \phi P_n = 0.95 \times 159 \text{ kips} )</td>
<td>For tension (overlapping) branch, ( P_n = 159 \text{ kips} )</td>
</tr>
<tr>
<td>( = 151 \text{ kips} )</td>
<td>( \frac{159 \text{ kips}}{1.58} = 101 \text{ kips} )</td>
</tr>
<tr>
<td>151 kips &gt; 150 kips ( \text{o.k.} )</td>
<td>101 kips &gt; 100 kips ( \text{o.k.} )</td>
</tr>
<tr>
<td>For compression (overlapped) branch, ( \phi P_n = 0.95 \times 248 \text{ kips} )</td>
<td>For compression (overlapped) branch, ( P_n = 248 \text{ kips} )</td>
</tr>
<tr>
<td>( = 236 \text{ kips} )</td>
<td>( \frac{248 \text{ kips}}{1.58} = 157 \text{ kips} )</td>
</tr>
<tr>
<td>236 kips &gt; 150 kips ( \text{o.k.} )</td>
<td>157 kips &gt; 100 kips ( \text{o.k.} )</td>
</tr>
</tbody>
</table>

Example 8.5—Gapped K-Connection with Square HSS and Unbalanced Branch Loads

Given:

A planar roof truss contains the planar HSS 45° gapped K-connection shown in Figure 8-10. Note that the chord moment is necessary for equilibrium because of the noding eccentricity. Because the vertical components of the branch member forces differ by more than 20%, the connection must be treated as a combination of two types: a K-connection and a cross-connection [see AISC Specification Section K2(b)], as will be demonstrated. The loads shown consist of live load and dead load in the ratio 3:1. Determine the adequacy of the HSS connection and also consider the design of the welds.

From AISC Manual Table 2-3, the HSS material properties are as follows:

- All members
- ASTM A500 Grade B
- \( F_y = 46 \text{ ksi} \)
- \( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-12, the HSS geometric properties are as follows:

\[
\begin{align*}
\text{HSS12×12×}\% & \\
A & = 25.7 \text{ in.}^2 \\
B & = 12.0 \text{ in.} \\
H & = 12.0 \text{ in.} \\
\tau & = 0.581 \text{ in.} \\
\text{HSS8×8×}\% & \\
A_{b} & = 10.4 \text{ in.}^2 \\
B_{b} & = 8.00 \text{ in.} \\
H_{b} & = 8.00 \text{ in.} \\
\tau_{b} & = 0.349 \text{ in.} \\
\end{align*}
\]

Fig. 8-10. Welded gap K-connection with square HSS.
Solution:

The total applied loads on each branch are:

\[
P_{\text{compression branch}} = 120 \text{ kips} + 40.0 \text{ kips} = 160 \text{ kips}
\]

\[
P_{\text{tension branch}} = 63.4 \text{ kips} + 21.2 \text{ kips} = 84.6 \text{ kips}
\]

From AISC Specification Section K2(b), when the punching load \((P, \sin \theta)\) in a branch member is essentially equilibrated (within 20%) by loads in other branch member(s) on the same side of the connection, the connection is classified as a K-connection. The relevant gap is between the primary branch members whose loads equilibrate. When the punching load \((P, \sin \theta)\) is transmitted through the chord member and is equilibrated by branch member(s) on the opposite side, the connection is classified as a cross-connection. When branch members transmit part of their load as K-connections and part of their load as T-, Y- or cross-connections, the adequacy of each branch is determined by linear interaction of the proportion of the branch load involved in each type of load transfer.

**Connection classification**

Based on the previous discussion, the classification of a connection as a K-connection is dependent on the following inequality:

\[
0.8 \leq \frac{P_{\text{compression branch}} \sin \theta_{\text{compression branch}}}{P_{\text{tension branch}} \sin \theta_{\text{tension branch}}} = \frac{160 \text{ kips} \sin 45^\circ}{84.6 \text{ kips} \sin 45^\circ} = 1.89 > 1.2
\]

Therefore, treat the connection as shown in Figure 8-11 (a combination of a K- and cross-connection).

**Analysis of K-Connection [Balanced; see Figure 8-11(a)]**

**Limits of applicability**

AISC Specification Section K2.3 and Table 8-2A provide the following limits of applicability for rectangular HSS.

\[-0.55 \leq e/H \leq 0.25\]

Based on a connection gap, \(g = 2.50\) in., and by geometry:

Eccentricity, \(e\)

\[
e = \frac{\sin \theta_1 \sin \theta_2}{\sin (\theta_1 + \theta_2)} \left( \frac{H_1}{2 \sin \theta_1} + \frac{H_2}{2 \sin \theta_2} + g \right) = \frac{H}{2}
\]

\[
e = \frac{(\sin 45^\circ)^2}{\sin 90^\circ} \left( \frac{8.00 \text{ in.}}{2 \sin 45^\circ} + \frac{8.00 \text{ in.}}{2 \sin 45^\circ} + 2.50 \text{ in.} \right) - \frac{12.0 \text{ in.}}{2}
\]

\[= 0.907 \text{ in.}\]

\[\begin{align*}
P_L &= 125 \text{ kips} \\
P_D &= 41.8 \text{ kips}
\end{align*}\]

\[\begin{align*}
P_L &= 63.4 \text{ kips} \\
P_D &= 21.2 \text{ kips}
\end{align*}\]

\[\begin{align*}
P_L &= 215 \text{ kips} \\
P_D &= 71.8 \text{ kips}
\end{align*}\]

\[\begin{align*}
P_L &= 63.4 \text{ kips} \\
P_D &= 21.2 \text{ kips}
\end{align*}\]

\[\begin{align*}
P_L &= 56.5 \text{ kips} \\
P_D &= 18.9 \text{ kips}
\end{align*}\]

\[\begin{align*}
P_L &= 40.0 \text{ kips} \\
P_D &= 13.3 \text{ kips}
\end{align*}\]

\[\begin{align*}
P_L &= 40.0 \text{ kips} \\
P_D &= 13.3 \text{ kips}
\end{align*}\]

Fig. 8-11. Loads for (a) balanced K-connection and (b) cross-connection.
Therefore

\[-0.55 \leq e/H = 0.0756 \leq 0.25 \quad \text{o.k.}\]

Because the noding eccentricity satisfies this limit, the resulting eccentricity moment that it produces \((M_L + M_D = 118 \text{ kip-in.} + 39.3 \text{ kip-in.} = 157 \text{ kip-in.})\) can be neglected with regard to connection design (however, it would still have an effect on chord member design in general).

\[
\theta_{b1} = \theta_{b2}
\]

\[
= 45^\circ \geq 30^\circ \quad \text{o.k.}
\]

\[
\frac{B}{t} = \frac{H}{t}
\]

\[
= 12.0 \text{ in.} \overline{0.581 \text{ in.}}
\]

\[
= 20.7 \leq 35 \quad \text{o.k}
\]

For the tension branch

\[
\frac{B_b}{t_b} = \frac{H_b}{t_b}
\]

\[
= 8.00 \text{ in.} \overline{0.349 \text{ in.}}
\]

\[
= 22.9 \leq 35 \quad \text{o.k.}
\]

For the compression branch

\[
\frac{B_b}{t_b} = \frac{H_b}{t_b}
\]

\[
= 8.00 \text{ in.} \overline{0.349 \text{ in.}}
\]

\[
= 22.9
\]

\[
\leq 1.25 \sqrt{\frac{E}{F_{yb}}} = 31.4 \quad \text{o.k.}
\]

\[
\frac{B_b}{t_b} = \frac{H_b}{t_b}
\]

\[
= 22.9 \leq 35 \quad \text{o.k.}
\]

\[
0.5 \leq \frac{H_b}{B_b} = \frac{H}{B} \leq 2.0
\]

\[
H_b = 12.0 \text{ in.}
\]

\[
B_b = 12.0 \text{ in.}
\]

\[
= 1.00
\]

\[
0.5 \leq 1.00 \leq 2.0 \quad \text{o.k.}
\]

\[
F_y = F_{yb}
\]

\[
= 46 \text{ ksi} \leq 52 \text{ ksi} \quad \text{o.k.}
\]

\[
\frac{F_y}{F_u} = \frac{F_{yb}}{F_{ub}}
\]

\[
= \frac{46 \text{ ksi}}{58 \text{ ksi}}
\]

\[
= 0.793 \leq 0.8 \quad \text{o.k.}
\]
$$\frac{B_b}{B} = \frac{H_b}{B} \geq 0.1 + \frac{\gamma}{50}$$

= 8.00 in. / 12.0 in.

= 0.667

$$0.1 + \frac{\gamma}{50} = 0.1 + \frac{(B/2t)}{50}$$

= 0.1 + \frac{12.0 \text{ in.} / (2)(0.581 \text{ in.})}{50}

= 0.307

0.667 \geq 0.307 \text{ o.k.}

$$\beta_{eff} = \frac{\text{sum of perimeter of two branch members}}{8B}$$

= \frac{8(8.00 \text{ in.})}{8(12.0 \text{ in.})}

= 0.667 \geq 0.35 \text{ o.k.}

$$\zeta = \frac{g}{B}$$

= \frac{2.50 \text{ in.}}{12.0 \text{ in.}}

= 0.208 \geq 0.5(1 - \beta_{eff})

0.5(1 - \beta_{eff}) = 0.5(1 - 0.667)

= 0.167

0.208 \geq 0.167 \text{ o.k.}

$$g = 2.50 \text{ in.} \geq t_b \text{ compression branch} + t_b \text{ tension}$$

\geq 0.349 \text{ in.} + 0.349 \text{ in.} = 0.698 \text{ in.} \text{ o.k.}

smaller B_b = 8.00 \text{ in.} \geq 0.63(\text{larger } B_b) = 5.04 \text{ in.} \text{ o.k.}

Because both branches are HSS 8\times8\times\frac{3}{8}, the smaller B_b equals the larger B_b.

Required strength for K-connection portion (expressed as a force in a branch)

From Chapter 2 of ASCE 7, the required strength for the compression and tension branch is:

<table>
<thead>
<tr>
<th>LRFD</th>
</tr>
</thead>
<tbody>
<tr>
<td>For compression branch and tension branch,</td>
</tr>
<tr>
<td>$$P_s = 1.2 \left(\frac{21.2 \text{ kips}}{}\right) + 1.6 \left(\frac{63.4 \text{ kips}}{}\right)$$</td>
</tr>
<tr>
<td>= 127 \text{ kips}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>For compression branch and tension branch,</td>
</tr>
<tr>
<td>$$P_s = 21.2 \text{ kips} + 63.4 \text{ kips}$$</td>
</tr>
<tr>
<td>= 84.6 \text{ kips}</td>
</tr>
</tbody>
</table>

Chord wall plastification

From AISC Specification Section K2.3c(a), the nominal strength for the limit state of chord wall plastification in gapped K-connections can be determined from the following:

$$P_s \sin \theta = F_{r,t} \left[9.8 \beta_{eff} \gamma^{0.5}\right] Q_f$$

(Spec. Eq. K2-20 and Table 8-2)
where
\[
\gamma = \frac{B}{2t} = 10.3
\]
\[
Q_f = 1.3 - 0.4 \frac{U}{\beta_{ef}} \leq 1.0 \text{ for chord in compression} \quad (\text{Spec. Eq. K2-11 and Table 8-2})
\]
\[
U = \left| \frac{P_c + M_e}{A_F c F_{cr}} \right| \quad (\text{Spec. Eq. K2-12 and Table 8-2})
\]

From Chapter 2 of ASCE 7, the required strength in the chord is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[P_i = P_a]</td>
<td>[P_i = P_a]</td>
</tr>
<tr>
<td>[= 1.2(71.8 \text{ kips}) + 1.6(215 \text{ kips})]</td>
<td>[= 71.8 \text{ kips} + 215 \text{ kips}]</td>
</tr>
<tr>
<td>[= 430 \text{ kips in chord on side of joint with higher compression stress}]</td>
<td>[= 287 \text{ kips in chord on side of joint with higher compression stress}]</td>
</tr>
<tr>
<td>[F_i = F_y]</td>
<td>[F_i = 0.6F_y]</td>
</tr>
<tr>
<td>[= 46 \text{ ksi}]</td>
<td>[= 27.6 \text{ ksi}]</td>
</tr>
<tr>
<td>[U = \left</td>
<td>\frac{430 \text{ kips}}{25.7 \text{ in.}^2(46 \text{ ksi})} \right</td>
</tr>
<tr>
<td>[Q_f = 1.3 - 0.4 \left( \frac{0.364}{0.667} \right)]</td>
<td>[Q_f = 1.3 - 0.4 \left( \frac{0.405}{0.667} \right)]</td>
</tr>
<tr>
<td>[= 1.08 \geq 1.0]</td>
<td>[= 1.06 \geq 1.0]</td>
</tr>
<tr>
<td>Use [Q_f = 1.0]</td>
<td>Use [Q_f = 1.0]</td>
</tr>
</tbody>
</table>

Therefore, the available strength for the limit state of chord wall plastification is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[P_n = 46 \text{ ksi} \left(0.581 \text{ in.}^2 \right) \left[9.8 \left(0.667\right)^2 \left(10.3\right)^{0.5} \right] \times 1.0/\sin 45^\circ]</td>
<td>[P_n = 46 \text{ ksi} \left(0.581 \text{ in.}^2 \right) \left[9.8 \left(0.667\right)^2 \left(10.3\right)^{0.5} \right] \times 1.0/\sin 45^\circ]</td>
</tr>
<tr>
<td>[= 461 \text{ kips}]</td>
<td>[= 461 \text{ kips}]</td>
</tr>
<tr>
<td>[\phi P_a = 0.90 \left(461 \text{ kips}\right)]</td>
<td>[\phi P_a = 461 \text{ kips}]</td>
</tr>
<tr>
<td>[= 415 \text{ kips}]</td>
<td>[\frac{P_n}{\bar{\Omega}} = \frac{461 \text{ kips}}{1.67} = 276 \text{ kips}]</td>
</tr>
<tr>
<td>for both branches since (\theta_b) is the same</td>
<td>for both branches since (\theta_b) is the same (\frac{84.6 \text{ kips}}{276 \text{ kips}} = 0.307)</td>
</tr>
<tr>
<td>Branch utilization = [\frac{127 \text{ kips}}{415 \text{ kips}} = 0.306]</td>
<td>Branch utilization = [\frac{127 \text{ kips}}{415 \text{ kips}} = 0.306]</td>
</tr>
</tbody>
</table>

All other limit states for gapped K-connections do not control (see AISC Specification Section K2.3c and Table 8-2) because the branches are square.
Analysis of Cross-Connection [see Figure 8-11(b)]

Limits of applicability

Note that only the compression branch needs to be checked, because this is the only branch that participates in the cross-connection force transfer. From AISC Specification Section K2.3a and Table 8-2A, the limits of applicability for this connection are:

\[
\theta_{m} = 45^\circ \geq 30^\circ \quad \text{O.K.}
\]

\[
\frac{B}{t} = \frac{H}{t} = 20.7 \leq 35 \quad \text{O.K.}
\]

\[
\frac{B_{b}}{t_{b}} = \frac{H_{b}}{t_{b}} = 22.9 \quad \text{for compression branch}
\]

\[
\leq 1.25 \sqrt{\frac{E}{F_{y}}} = 31.4 \quad \text{O.K.}
\]

\[
\frac{B_{b}}{t_{b}} = \frac{H_{b}}{t_{b}} = 22.9 \leq 35 \quad \text{O.K.}
\]

\[
\frac{B_{b}}{B} = \frac{H_{b}}{H} \leq 0.667 \geq 0.25 \quad \text{O.K.}
\]

\[
0.5 \leq \frac{H_{b}}{B_{b}} = \frac{H}{B} = 1.00 \leq 2.0 \quad \text{O.K.}
\]

\[
F_{y} = F_{y} \quad \text{ksi} \leq 52 \quad \text{O.K.}
\]

\[
\frac{F_{y}}{F_{y}} = \frac{F_{u}}{F_{u}} = 0.793 \leq 0.8 \quad \text{O.K.}
\]

Required strength for the cross-connection portion (expressed as a force in the branch)

From Chapter 2 of ASCE 7, the required strength is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>For the compression branch, (P_{u} = 1.2(18.9 \text{ kips}) + 1.6(56.5 \text{ kips}) = 113 \text{ kips})</td>
<td>For the compression branch, (P_{u} = 18.9 \text{ kips} + 56.5 \text{ kips} = 75.4 \text{ kips})</td>
</tr>
</tbody>
</table>

Limit state of chord wall plastification

Because for the compression branch, \(\beta = B_{b}/B = 0.667 \leq 0.85\), the limit state of chord wall plastification is applicable. From AISC Specification Section K2.3b(a), the nominal strength can be determined from the following:

\[
P_{u} \sin \theta = F_{t} t^{*} \left[ \frac{2\eta}{(1-\beta)} + \frac{4}{\sqrt{1-\beta}} \right] Q_{f}
\]

\((\text{Spec. Eq. K2-13 and Table 8-2})\)
where

\[ \beta \text{ for the compression branch} = \frac{B_b}{B} \]

\[ = 0.667 \leq 0.85 \]

\[ \eta = \frac{H_b}{B \sin \theta} \]

\[ = \frac{8.00 \text{ in.}}{12.0 \text{ in.} \sin 45^\circ} \]

\[ = 0.943 \]

\[ Q_f = 1.3 - 0.4 \frac{U}{\beta} \leq 1.0 \text{ for chord in compression} \]

\[ U = \frac{P_c}{AF_c + M_c/SF_c} \]

(Spec. Eq. K2-10 and Table 8-2)

From Chapter 2 of ASCE 7, the required strength in the chord is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_r )</td>
<td>1.2(13.3 kips) + 1.6(40.0 kips)</td>
<td>( P_r ) = ( P_u ) = 13.3 kips + 40.0 kips</td>
</tr>
<tr>
<td></td>
<td>= 80.0 kips in chord on side of joint with higher compression stress</td>
<td>= 53.3 kips in chord on side of joint with higher compression stress</td>
</tr>
<tr>
<td>( M_r )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( F_r )</td>
<td>( F_y )</td>
<td>( F_c = 0.6F_y ) = 27.6 ksi</td>
</tr>
<tr>
<td>( U )</td>
<td>0.0677</td>
<td>0.0751</td>
</tr>
</tbody>
</table>
| \( Q_f \)    | \( 1.3 - 0.4 \left( \frac{0.0677}{0.667} \right) \) 1.26 \geq 1.0 | \( Q_f = 1.3 - 0.4 \left( \frac{0.0751}{0.667} \right) \) \( \geq 1.25 \)
| Use \( Q_f \) | 1.0  | 1.0  |

Therefore, the available strength for the limit state of chord wall plastification is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_n )</td>
<td>( 46 \text{ ksi} \left( 0.581 \text{ in.} \right)^2 \times \left[ \frac{2(0.943)}{(1-0.667)} + \frac{4}{\sqrt{1-0.667}} \right] \frac{1.0}{\sin 45^\circ} ) 277 kips</td>
<td>( P_n ) = ( 46 \text{ ksi} \left( 0.581 \text{ in.} \right)^2 \times \left[ \frac{2(0.943)}{(1-0.667)} + \frac{4}{\sqrt{1-0.667}} \right] \frac{1.0}{\sin 45^\circ} ) 277 kips</td>
</tr>
<tr>
<td>( \phi P_n )</td>
<td>1.00(277 kips)</td>
<td>( \phi P_n ) = 1.00(277 kips)</td>
</tr>
<tr>
<td>( \phi P_n )</td>
<td>277 kips</td>
<td>( \phi P_n ) = 277 kips</td>
</tr>
</tbody>
</table>
| Compression branch utilization | \( \frac{113 \text{ kips}}{277 \text{ kips}} = 0.408 \) | Compression branch utilization = \( \frac{75.4 \text{ kips}}{185 \text{ kips}} = 0.408 \)
The limit states that are applicable for $1.0 \geq \beta \geq 0.85$ do not control and need not be checked because $\beta = 0.667$. The limit state of shearing of the chord sidewalls is not applicable as there is no shear plane evident. (The flange of the W8×24 is wider than the connection gap, $g = 2.50$ in.)

**Total utilization of each branch member**

For the compression branch: $0.31$ (Balanced K) + $0.41$ (Cross) = $0.72 \leq 1.0$ o.k.
For the tension branch: $0.31$ (Balanced K) + $0$ (Cross) = $0.31 \leq 1.0$ o.k.
(Summations—in this case—are the same for both LRFD and ASD.)

**Weld design considerations**

The compression branch, inclined at $\theta = 45^\circ$ behaves partly as a gapped K-connection branch and partly as a cross-connection branch. For $\theta \leq 50^\circ$, all four sides of a gapped K-connection perimeter weld are effective, whereas only three sides of a cross-connection perimeter weld are effective (AISC Specification Section K2.3e). Because the two branches are the same size, it would hence be conservative to assume a total effective weld length, around one branch, as

$$L_e = \frac{2(H_b - 1.2t_h)}{\sin \theta} + \left(1.2B_b - 1.2t_h\right)$$

$\left(\text{Spec. Eq. K2-29}\right)$

$$L_e = \frac{2\left[8.00 \text{ in.} - 1.2(0.349 \text{ in.})\right]}{\sin 45^\circ} + \left[8.00 \text{ in.} - 1.2(0.349 \text{ in.})\right]$$

$$L_e = 29.0 \text{ in.}$$

The weld can then be designed in accordance with procedures given in Chapter 2 of this Design Guide and in AISC Design Guide No. 21, Welded Connections—A Primer for Engineers (Miller, 2006). Alternatively, as noted in the AISC Specification Commentary to Chapter K (and also in the User Note in AISC Specification Section K1.3b), welds may be proportioned to develop the capacity of the connected branch wall, at all points along the weld length. Welds sized in this latter manner represent an upper limit on the required weld size, for any loading condition.
Chapter 9
HSS-to-HSS Moment Connections

9.1 SCOPE AND BASIS

The scope of this chapter follows the AISC Specification Section K3 and is limited to planar connections between HSS (or box members) directly welded to one another, in the form of T-, Y- (for round HSS only), and cross-connections. T- and Y-connections have one branch welded to a continuous chord (T-connections having a 90° branch angle), and cross-connections have a branch on each (opposite) side of a continuous chord. For the design criteria herein, the centerlines of the branch(es) and the chord member all lie in a common plane. Round HSS-to-round HSS and rectangular HSS-to-rectangular HSS are the only combinations of HSS shapes considered. The connections may be subject to branch in-plane bending moments, branch out-of-plane bending moments, or a combination of branch bending plus axial load. This chapter is thus applicable to frames with partially rigid (PR) or fully rigid (FR) moment connections, such as Vierendeel girders. The provisions in this chapter are not generally applicable to typical planar triangulated trusses (which are covered in Chapter 8) because the latter should be analyzed in a manner that results in no bending moments in the web members (see Section 8.4). Thus, K-connections with moment loading on the branches are not covered by this Design Guide.

Pertinent design criteria for round HSS-to-HSS T-, Y- and cross-connections are succinctly tabulated (Table 9-1), followed by a design example on such a connection. Similarly, pertinent design criteria for rectangular (which includes square) HSS-to-HSS T- and cross-connections are tabulated (Table 9-2), followed by two design examples on such connections. For connection configurations beyond the scope of this chapter, one can refer to other authoritative design guidance such as AWS D1.1 (AWS, 2008).

The design criteria in this chapter and the AISC Specification Section K3 are based on failure modes, or limit states, that have been reported in international research on HSS. In particular, the basis for the equations in AISC Specification Section K3, and also this chapter, is Eurocode 3 (CEN, 2005). This represents one of the most up-to-date consensus specifications or recommendations on welded HSS-to-HSS connections. The equations used in AISC Specification Section K3 and in this chapter have also been adopted in CIDECT Design Guide No. 9 (Kurobane et al., 2004).

9.2 NOTATION AND LIMIT STATES

Some common notation associated with branch and chord members in welded HSS connections has been illustrated in Figure 8-1. Available testing for HSS-to-HSS moment connections is much less extensive than that for axially loaded T-, Y-, cross- and K-connections. Hence, the governing limit states to be checked for axially loaded connections have been used as a basis for the possible limit states in moment-loaded connections. Thus, the design criteria for round HSS moment connections are based on the limit states of chord plastification and shear yielding (punching) failure, with $F$ and $\Omega$ factors consistent with AISC Specification Section K3.2. For rectangular HSS moment connections, the design criteria are based on the limit states of chord wall plastification, sidewall local yielding, local yielding due to uneven load distribution, and chord distortional failure, with $F$ and $\Omega$ factors consistent with AISC Specification Section K3.3. The chord distortional failure mode is applicable only to rectangular HSS T- or cross-connections with an out-of-plane bending moment on the branch (or branches) causing a torque on the chord cross-section. This limit state of the chord cross-section need not be checked for T-connections if rhomboidal distortion of the chord is prevented (e.g., by the use of stiffeners or diaphragms to maintain the rectangular shape of the chord). Chord distortional failure also need not be checked for balanced cross-connections (where the branch moments are in equilibrium). Potential failure modes for HSS moment connections, illustrated for a rectangular HSS T-connection under in-plane branch bending, are given in Figure 9-1.

It is important to note that a number of potential limit states can often be precluded from the connection checking procedure because the corresponding failure modes are excluded by virtue of the connection geometry and the constrained limits of applicability of various parameters. The limits of applicability of the equations in this chapter (and in AISC Specification Section K3), given in Tables 9-1A and 9-2A, are predominantly reproduced from AISC Specification Section K2.
(a) Chord wall plastification.

(b) Shear yielding (punching) of the chord.

(c) Local yielding of tension side of branch, due to uneven load distribution.

(d) Local yielding of compression side of branch, due to uneven load distribution.

(e) Shear of the chord sidewalls.

(f) Chord sidewall failure.

Fig. 9-1. Potential limit states for HSS-to-HSS moment connections.
### 9.3 CONNECTION NOMINAL CAPACITY TABLES

#### Table 9-1. Nominal Capacities of Round HSS-to-HSS Moment Connections

<table>
<thead>
<tr>
<th>Connection Type</th>
<th>Connection Nominal Moment Capacity*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch(es) under In-Plane Bending</td>
<td>Limit State: Chord Plastification</td>
</tr>
</tbody>
</table>
| T-, Y- and Cross-Connections | \( M_r \sin \theta = 5.39 F_t t^2 \gamma^{0.5} \beta D_b Q_t \)  
\( \phi = 0.90 \text{ (LRFD)} \)  
\( \Omega = 1.67 \text{ (ASD)} \)  
| Limit State: Shear Yielding (Punching), when \( D_b \leq (D - 2t) \) |  
\( M_n = 0.6 F_t t D_b \left( 1 + 3 \sin \theta \right) \left( \frac{1}{4 \sin^2 \theta} \right) \)  
\( \phi = 0.95 \text{ (LRFD)} \)  
\( \Omega = 1.58 \text{ (ASD)} \)  
| Branch(es) under Out-of-Plane Bending | Limit State: Chord Plastification |
| T-, Y- and Cross-Connections | \( M_r \sin \theta = F_t t^2 D_b \)  
\( \phi = 0.90 \text{ (LRFD)} \)  
\( \Omega = 1.67 \text{ (ASD)} \)  
| Limit State: Shear Yielding (Punching), when \( D_b \leq (D - 2t) \) |  
\( M_n = 0.6 F_t t D_b \left( 3 + \sin \theta \right) \left( \frac{1}{4 \sin^2 \theta} \right) \)  
\( \phi = 0.95 \text{ (LRFD)} \)  
\( \Omega = 1.58 \text{ (ASD)} \)  

For T-, Y- and cross-connections, with branch(es) under combined axial load, in-plane bending and out-of-plane bending, or any combination of these load effects:

**LRFD:** \( \left( \frac{P_r}{\phi P_r} \right) + \left( \frac{M_{r,\phi}}{\phi M_{r,\phi}} \right)^2 + \left( \frac{M_{n,\phi}}{\phi M_{n,\phi}} \right) \leq 1.0 \)  
from (K3-7)

**ASD:** \( \left( \frac{P_r}{P_a/\Omega} \right) + \left[ \frac{M_{r,\phi}}{M_{r,\phi}/\Omega} \right]^2 + \left[ \frac{M_{n,\phi}}{M_{n,\phi}/\Omega} \right] \leq 1.0 \)  
from (K3-8)

\( \phi P_r = \text{design strength (or } P_r/\Omega = \text{allowable strength) obtained from Table 8-1 or AISC Specification Section K2.2b} \)

\( \phi M_{r,\phi} = \text{design strength (or } M_{r,\phi}/\Omega = \text{allowable strength) for in-plane bending (above or AISC Specification Section K3.2b)} \)

\( \phi M_{n,\phi} = \text{design strength (or } M_{n,\phi}/\Omega = \text{allowable strength) for out-of-plane bending (above or AISC Specification Section K3.2c)} \)

**Functions**

\( Q_t = 1 \) for chord (connecting surface) in tension

\( Q_t = 1.0 - 0.3 U (1 + U) \) for chord (connecting surface) in compression  
\( \text{(K3-1)} \)

\[ U = \left| \frac{P_r}{AF_c} + \frac{M_r}{SF_t} \right| \]  
\( \text{where } P_r \text{ and } M_r \text{ refer to the required axial and flexural strength in the chord: } P_r = P_a \text{ for LRFD}; P_s \text{ for ASD.} \)

\( M_r = M_t \text{ for LRFD; } M_s \text{ for ASD.} \)

* Equation references are to the AISC Specification.
Table 9-1A. Limits of Applicability of Table 9-1

<table>
<thead>
<tr>
<th>Limit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch angle:</td>
<td>$\theta \geq 30^\circ$</td>
</tr>
<tr>
<td>Chord wall slenderness:</td>
<td>$D/t \leq 50$ for T- and Y-connections</td>
</tr>
<tr>
<td></td>
<td>$D/t \leq 40$ for cross-connections</td>
</tr>
<tr>
<td>Branch wall slenderness:</td>
<td>$D_t/t_b \leq 50$</td>
</tr>
<tr>
<td></td>
<td>$D_t/t_b \leq 0.05E/F_{y0}$</td>
</tr>
<tr>
<td>Width ratio:</td>
<td>$0.2 &lt; D_i/D \leq 1.0$</td>
</tr>
<tr>
<td>Material strength:</td>
<td>$F_y$ and $F_{y0} \leq 52$ ksi (360 MPa)</td>
</tr>
<tr>
<td>Ductility:</td>
<td>$F_y/F_u$ and $F_{y0}/F_{u0} \leq 0.8$</td>
</tr>
</tbody>
</table>

Note: Limits of applicability are from AISC Specification Section K3.2a.

Table 9-2. Nominal Capacities of Rectangular HSS-to-HSS Moment Connections

<table>
<thead>
<tr>
<th>Connection Type</th>
<th>Connection Nominal Moment Capacity*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch(es) under In-Plane Bending</td>
<td>Limit State: Chord Wall Plastification, when $\beta \leq 0.85$</td>
</tr>
<tr>
<td>T- and Cross-Connections</td>
<td>$M_{n} = F_y t^2 H_b \left[ 1 + \frac{2}{2\eta} \sqrt{1 - \frac{\eta}{(1 - \beta)}} \right] Q_i$  [(K3-11)]</td>
</tr>
<tr>
<td></td>
<td>$\phi = 1.00$ (LRFD) $\Omega = 1.50$ (ASD)</td>
</tr>
<tr>
<td></td>
<td>Limit State: Sidewall Local Yielding, when $\beta \geq 0.85$</td>
</tr>
<tr>
<td></td>
<td>$M_{n} = 0.5F_y t^2 (H_b + 5t)^2$ [(K3-12)]</td>
</tr>
<tr>
<td></td>
<td>$\phi = 1.00$ (LRFD) $\Omega = 1.50$ (ASD)</td>
</tr>
<tr>
<td>Limit State: Local Yielding of Branch/Branches Due to Uneven Load Distribution, when $\beta \geq 0.85$</td>
<td>$M_{n} = F_{yb} \left[ Z_b - \left( 1 - \frac{b_{out}}{B_c} \right) B_c H_t t_b \right]$ [(K3-13)]</td>
</tr>
<tr>
<td></td>
<td>$\phi = 0.95$ (LRFD) $\Omega = 1.58$ (ASD)</td>
</tr>
<tr>
<td>Branch(es) under Out-of-Plane Bending</td>
<td>Limit State: Chord Wall Plastification, when $\beta \leq 0.85$</td>
</tr>
<tr>
<td>T- and Cross-Connections</td>
<td>$M_{n} = F_y t^2 \left[ 0.5H_b (1 + \beta) \frac{2BB_b (1 + \beta)}{(1 - \beta)} \right] Q_i$ [(K3-15)]</td>
</tr>
<tr>
<td></td>
<td>$\phi = 1.00$ (LRFD) $\Omega = 1.50$ (ASD)</td>
</tr>
<tr>
<td></td>
<td>Limit State: Sidewall Local Yielding, when $\beta \geq 0.85$</td>
</tr>
<tr>
<td></td>
<td>$M_{n} = F_y t (B - t) (H_b + 5t)$ [(K3-16)]</td>
</tr>
<tr>
<td></td>
<td>$\phi = 1.00$ (LRFD) $\Omega = 1.50$ (ASD)</td>
</tr>
<tr>
<td>Limit State: Local Yielding of Branch/Branches Due to Uneven Load Distribution, when $\beta \geq 0.85$</td>
<td>$M_{n} = F_{yb} \left[ Z_b - 0.5 \left( 1 - \frac{b_{out}}{B_c} \right) ^2 B_c^2 t_b \right]$ [(K3-17)]</td>
</tr>
<tr>
<td></td>
<td>$\phi = 0.95$ (LRFD) $\Omega = 1.58$ (ASD)</td>
</tr>
<tr>
<td>Limit State: Chord Distortional Failure, for T-Connections and Unbalanced Cross-Connections</td>
<td>$M_{n} = 2F_y t \left[ H_t t + \sqrt{BHt (B + H)} \right]$ [(K3-19)]</td>
</tr>
<tr>
<td></td>
<td>$\phi = 1.00$ (LRFD) $\Omega = 1.50$ (ASD)</td>
</tr>
</tbody>
</table>
Table 9-2 (continued)

For T- and cross-connections, with branch(es) under combined axial load, in-plane bending and out-of-plane bending, or any combination of these load effects:

\[
\begin{align*}
\text{LRFD: } & \left( \frac{P_r}{\phi P_a} \right) + \left( \frac{M_{r,\phi}}{\phi M_{a,\phi}} \right) + \left( \frac{M_{r,\phi}}{\phi M_{a,\phi}} \right) \leq 1.0 \quad (K3-20) \\
\text{ASD: } & \left( \frac{P_a}{P_u} \right) + \left( \frac{M_{a,\phi}}{M_{u,\phi}} \right) + \left( \frac{M_{a,\phi}}{M_{u,\phi}} \right) \leq 1.0 \quad (K3-21)
\end{align*}
\]

\( \phi P_r \) = design strength (or \( P_r / \phi P_a \) = allowable strength) obtained from Table 8-2 or AISC Specification Section K2.3b

\( \phi M_{r,\phi} \) = design strength (or \( M_{r,\phi} / \phi M_{a,\phi} \) = allowable strength) for in-plane bending (above or AISC Specification Section K3.3b)

\( \phi M_{r,\phi} \) = design strength (or \( M_{r,\phi} / \phi M_{a,\phi} \) = allowable strength) for out-of-plane bending (above or AISC Specification Section K3.3c)

**Functions**

\[ Q_f = 1 \quad \text{for chord (connecting surface) in tension} \]

\[ Q_f = 1.3 - 0.4 \frac{U}{P} \leq 1.0 \quad \text{for chord (connecting surface) in compression} \quad (K3-9) \]

\[ U = \left| \frac{P_r}{AF_c} + \frac{M_r}{SF_r} \right| \]

where \( P_r \) and \( M_r \) refer to the required axial and flexural strength in the chord: \( P_r = P_u \) for LRFD; \( P_a \) for ASD. \( M_r = M_u \) for LRFD; \( M_a \) for ASD.

\[ F^*_y = F_y \quad \text{for T-connections and } = 0.8 F_y \text{ for cross-connections} \]

\[ b_{\text{eff}} = \frac{10}{B/t} \left( \frac{F^*_y}{F_{y,\phi}} \right) B_0 \leq B_0 \quad (K3-14) \]

* Equation references are to the AISC Specification.

Table 9-2A. Limits of Applicability of Table 9-2

<table>
<thead>
<tr>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch angle:</td>
</tr>
<tr>
<td>Chord wall slenderness:</td>
</tr>
<tr>
<td>Branch wall slenderness:</td>
</tr>
<tr>
<td>Width ratio:</td>
</tr>
<tr>
<td>Aspect ratio:</td>
</tr>
<tr>
<td>Material strength:</td>
</tr>
<tr>
<td>Ductility:</td>
</tr>
</tbody>
</table>

Note: Limits of applicability are from AISC Specification Section K3.3a.
9.4 CONNECTION DESIGN EXAMPLES

Example 9.1—Cross-Connection with Round HSS (In-Plane Bending)

Given:

Determine the adequacy of the welded HSS cross-connection under the member loads shown in Figure 9-2. Loads consist of 25% dead load and 75% live load. Assume the welds are strong enough to develop the yield strength of the connected branch wall at all locations around the branch.

From AISC Manual Table 2-3, the material properties are as follows:

- All members
  - ASTM A500 Grade B
  - \( F_y = 42 \text{ ksi} \)
  - \( F_u = 58 \text{ ksi} \)

From AISC Manual Table 1-13, the HSS geometric properties are as follows:

- HSS 6.000\times 0.375
  - \( A = 6.20 \text{ in.}^2 \)
  - \( D = 6.00 \text{ in.} \)
  - \( t = 0.349 \text{ in.} \)

- HSS 4.000\times 0.250
  - \( A_b = 2.76 \text{ in.}^2 \)
  - \( D_b = 4.00 \text{ in.} \)
  - \( t_b = 0.233 \text{ in.} \)

Solution:

Limits of applicability

From AISC Specification Section K3.2a and Table 9-1A, the limits of applicability for round HSS are:

- \( \theta = 45^\circ \geq 30^\circ \)  \( \text{o.k.} \)
- \( \frac{D}{t} = \frac{6.00 \text{ in.}}{0.349 \text{ in.}} = 17.2 \leq 40 \)  \( \text{o.k.} \)

![Fig. 9-2. Welded cross-connection with round HSS.](image)
\[ \frac{D_h}{t_b} = \frac{4.00 \text{ in.}}{0.233 \text{ in.}} \]
\[ = 17.2 \leq 50 \quad \text{o.k.} \]

\[ \frac{D_h}{D} = \frac{4.00 \text{ in.}}{6.00 \text{ in.}} \]
\[ = 0.667 \]
\[ 0.2 < \frac{D_h}{D} \leq 1.0 \]

\[ \frac{D_h}{D} = 4.00 \text{ in.} \]
\[ = 0.667 \]
\[ 0.2 < 0.667 \leq 1.0 \quad \text{o.k.} \]

\[ F_y = F_{sh} \]
\[ = 42 \text{ ksi} \leq 52 \text{ ksi} \quad \text{o.k.} \]

\[ \frac{F_y}{F_{u}} = \frac{F_{sh}}{F_{sh}} \leq 0.8 \]
\[ \frac{F_y}{F_{u}} = \frac{46 \text{ ksi}}{58 \text{ ksi}} \]
\[ = 0.724 \leq 0.8 \quad \text{o.k.} \]

**Required flexural strength (expressed as an in-plane bending moment in the branch)**

From Chapter 2 of ASCE 7, the required flexural strength of the connection is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_u = 1.2(30.0 \text{ kip-in.}) + 1.6(90.0 \text{ kip-in.}) )</td>
<td>( M_u = 30.0 \text{ kip-in.} + 90.0 \text{ kip-in.} )</td>
</tr>
<tr>
<td>( = 180 \text{ kip-in.} )</td>
<td>( = 120 \text{ kip-in.} )</td>
</tr>
</tbody>
</table>

**Chord plastification**

The nominal flexural strength for the limit state of chord plastification, for cross-connections with in-plane bending moments in the branches, is:

\[ M_u \sin \theta = 5.39 F_i \gamma^{0.3} \beta D_o Q_f \quad \text{(Spec. Eq. K3-3 and Table 9-1)} \]
where
\[
\beta = \frac{D}{D_b} = \frac{4.00 \text{ in.}}{6.00 \text{ in.}} = 0.667
\]
\[
\gamma = \frac{D}{2t} = \frac{6.00 \text{ in.}}{2(0.349 \text{ in.})} = 8.60
\]
\[
Q_f = 1.0 - 0.3U(1+U)
\]
\[
U = \frac{P_r + M_t F_y}{A F_r S F_c}
\]
for chord in compression  
(Spec. Eq. K3-1 and Table 9-1)

From Chapter 2 of ASCE 7, the required axial strength, \( P_r \), in the chord is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_r = P_a ) ( = 1.2(25.0 \text{ kips}) + 1.6(75.0 \text{ kips}) ) ( = 150 \text{ kips in chord} )</td>
<td>( P_r = P_a ) ( = 25.0 \text{ kips} + 75.0 \text{ kips} ) ( = 100 \text{ kips in chord} )</td>
</tr>
<tr>
<td>( M_t = 0 )</td>
<td>( M_t = 0 )</td>
</tr>
<tr>
<td>( F_c = F_y ) ( = 42 \text{ ksi} )</td>
<td>( F_c = 0.6F_y ) ( = 25.2 \text{ ksi} )</td>
</tr>
<tr>
<td>( U = \frac{150 \text{ kips}}{6.20 \text{ in.}^2(42 \text{ ksi})} ) ( = 0.576 )</td>
<td>( U = \frac{100 \text{ kips}}{6.20 \text{ in.}^2(25.2 \text{ ksi})} ) ( = 0.640 )</td>
</tr>
<tr>
<td>( Q_f = 1.0 - 0.3(0.576)(1+0.576) ) ( = 0.728 )</td>
<td>( Q_f = 1.0 - 0.3(0.640)(1+0.640) ) ( = 0.685 )</td>
</tr>
</tbody>
</table>

The available flexural strength of the connection is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_n = 5.39(42 \text{ ksi})(0.349 \text{ in.})^2(8.60)^{0.5} \times (0.667)(4.00 \text{ in.})(0.728)/\sin45^\circ ) ( = 222 \text{ kip-in.} )</td>
<td>( M_n = 5.39(42 \text{ ksi})(0.349 \text{ in.})^2(8.60)^{0.5} \times (0.667)(4.00 \text{ in.})(0.685)/\sin45^\circ ) ( = 209 \text{ kip-in.} )</td>
</tr>
<tr>
<td>( \phi M_n = 0.90(222 \text{ kip-in.}) ) ( = 200 \text{ kip-in.} )</td>
<td>( M_n = 209 \text{ kip-in.} ) ( \frac{M_n}{\Omega} = 1.67 ) ( = 125 \text{ kip-in.} )</td>
</tr>
<tr>
<td>200 kip-in. &gt; 180 kip-in. ( \text{o.k.} )</td>
<td>125 kip-in. &gt; 120 kip-in. ( \text{o.k.} )</td>
</tr>
</tbody>
</table>
Limit state of shear yielding (punching)

According to AISC Specification Section K3.2b(b), the limit state of shear yielding (punching) need not be checked when \( \beta > (1 - 1/\gamma) \). Because \( \beta \leq (1 - 1/\gamma) \), which reduces to \( D_b = 4.00 \text{ in.} < (D - 2t) = 5.30 \text{ in.} \), this limit state must be considered. From AISC Specification Section K3.2b(b), the nominal strength is:

\[
M_n = 0.6F_t D_b^2 \left[ \frac{1 + 3 \sin \theta}{4 \sin^2 \theta} \right] = 0.6 \left( 42 \text{ ksi} \right) \left( 0.349 \text{ in.} \right) \left( 4.00 \text{ in.} \right) \left[ \frac{1 + 3 \sin 45^\circ}{4 \sin^2 45^\circ} \right] = 220 \text{ kip-in.}
\]

The available flexural strength of the connection is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi M_n = 0.95 \left( 220 \text{ kip-in.} \right) )</td>
<td></td>
</tr>
<tr>
<td>= 209 kip-in.</td>
<td></td>
</tr>
<tr>
<td>209 kip-in. &gt; 180 kip-in. o.k.</td>
<td></td>
</tr>
<tr>
<td>( M_n = \frac{220 \text{ kip-in.}}{1.58} )</td>
<td></td>
</tr>
<tr>
<td>= 139 kip-in.</td>
<td></td>
</tr>
<tr>
<td>139 kip-in. &gt; 120 kip-in. o.k.</td>
<td></td>
</tr>
</tbody>
</table>

Example 9.2—Vierendeel Connection with Square HSS (In-Plane Bending)

Given:

A planar Vierendeel frame has an interior connection subject to the loads shown in Figure 9-3. The loads indicated consist of 50% dead load and 50% live load. For a fully rigid connection, it is recommended that such a connection have a width ratio, \( \beta = 1.0 \), and a chord overall width-to-thickness ratio not exceeding 16 (Packer and Henderson, 1997). It can be seen that these conditions are fulfilled with these selected members. Determine the adequacy of the connection under these in-plane branch loads. Assume the welds are strong enough to develop the yield strength of the connected branch wall at all locations around the branch.

From AISC Manual Table 2-3, the material properties are as follows:

Both members
ASTM A500 Grade B
\( F_y = 46 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

From AISC Manual Table 2-3, the material properties are as follows:

Both members
ASTM A500 Grade B
\( F_y = 46 \text{ ksi} \)
\( F_u = 58 \text{ ksi} \)

\[
M_t = 165 \text{ kip-in.} \quad M_D = 165 \text{ kip-in.}
\]

\[
V_t = 82.5 \text{ kip-in.} \quad V_D = 82.5 \text{ kip-in.}
\]

\[
P_t = 2.87 \text{ kips} \quad P_D = 2.87 \text{ kips}
\]

\[
V_t = 2.39 \text{ kips} \quad V_D = 2.39 \text{ kips}
\]

\[
P_t = 7.45 \text{ kips} \quad P_D = 7.45 \text{ kips}
\]

\[
V_t = 1.44 \text{ kips} \quad V_D = 1.44 \text{ kips}
\]

\[
M_t = 82.5 \text{ kip-in.} \quad M_D = 82.5 \text{ kip-in.}
\]

\[
V_t = 4.89 \text{ kips} \quad V_D = 4.89 \text{ kips}
\]

\[
P_t = 9.55 \text{ kips} \quad P_D = 9.55 \text{ kips}
\]

Fig. 9-3. Vierendeel connection with square HSS.
From AISC Manual Table 1-12, the HSS geometric properties are as follows:

**HSS6×6×½**

- $A = 9.74$ in.$^2$
- $B = 6.00$ in.
- $H = 6.00$ in.
- $t = 0.465$ in.
- $S = 16.1$ in.$^3$

**HSS6×6×⅙**

- $A_b = 6.43$ in.$^2$
- $B_b = 6.00$ in.
- $H_b = 6.00$ in.
- $t_b = 0.291$ in.
- $Z_b = 13.6$ in.$^3$

**Solution:**

*Limits of applicability for axial and moment loading*

From AISC Specification Section K2.3a and K3.3a, as well as Tables 8-2A and 9-2A, the following limits of applicability must be satisfied in the connection when the branch is loaded both axially and by bending moments:

- $\theta = 90^\circ \geq 30^\circ$ (Table 8-2A) and $\leq 90^\circ$ (Table 9-2A) \(\text{o.k.}\)

The following limits of applicability are from both Tables 8-2A and 9-2A:

- \(\frac{B}{t} = \frac{H}{t} = \frac{6.00\text{ in.}}{0.465\text{ in.}} = 12.9 \leq 35 \text{ o.k.}\)
- \(\frac{H_b}{t_b} = \frac{B_b}{t_b} \leq 35\)
- \(\frac{H_b}{t_b} = \frac{6.00\text{ in.}}{0.291\text{ in.}} = 20.6 \leq 35 \text{ o.k.}\)
- \(\frac{H_b}{t_b} = \frac{B_b}{t_b} \leq 1.25 \sqrt{\frac{E}{F_{ob}}} = \frac{1.25 \sqrt{29,000\text{ ksi}}}{46\text{ ksi}} = 31.4\)
- \(\frac{H_b}{t_b} = 20.6 \leq 31.4 \text{ o.k.}\)
- \(\frac{B_b}{B} = \frac{H_b}{B} = \frac{6.00\text{ in.}}{6.00\text{ in.}} = 1.00 \geq 0.25 \text{ o.k.}\)
- \(0.5 \leq \frac{H_b}{B_b} = \frac{H}{B} = 1.00 \leq 2.0 \text{ o.k.}\)
\[ F_y = F_{yb} \]
\[ = 46 \text{ ksi} \leq 52 \text{ ksi} \quad \text{o.k.} \]
\[ \frac{F_y}{F_{ub}} = \frac{F_{yb}}{F_{ub}} \]
\[ = \frac{46 \text{ ksi}}{58 \text{ ksi}} \]
\[ = 0.793 \leq 0.8 \quad \text{o.k.} \]

**Required axial strength (expressed as a force in the branch)**

From Chapter 2 of ASCE 7, the required strength of the connection due to the axial load in the branch is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u = 1.2 \left( 0.955 \text{ kips} \right) + 1.6 \left( 0.955 \text{ kips} \right) )</td>
<td>( P_u = 0.955 \text{ kips} + 0.955 \text{ kips} )</td>
</tr>
<tr>
<td>= 2.67 kips</td>
<td>= 1.91 kips</td>
</tr>
</tbody>
</table>

**Required moment capacity (expressed as a moment in the branch)**

From Chapter 2 of ASCE 7, the required strength of the connection due to the moment in the branch is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_u = 1.2 \left( 165 \text{ kip-in.} \right) + 1.6 \left( 165 \text{ kip-in.} \right) )</td>
<td>( M_u = 165 \text{ kip-in.} + 165 \text{ kip-in.} )</td>
</tr>
<tr>
<td>= 462 kip-in.</td>
<td>= 330 kip-in.</td>
</tr>
</tbody>
</table>

**Analysis under Branch Axial Loading**

**Local yielding of chord sidewalls**

From AISC Specification Section K2.3b(c), the limit state of local yielding of the chord sidewalls must be checked if \( \beta = 1 \).

Because \( \beta = B_s/B = 6.00 \text{ in.}/6.00 \text{ in.} = 1 \), the nominal strength of the branch for this limit state is determined from:

\[ P_{\sin \theta} = 2F_y t \left( 5k + N \right) \quad \text{(Spec. Eq. K2-15 and Table 8-2)} \]

where

- \( k \) = outside corner radius of HSS
- \( k = 1.5t \)
- \( = 0.698 \text{ in.} \)
- \( N \) = bearing length on chord
- \( = 6.00 \text{ in.} \)

Therefore, the nominal strength is

\[ P_{\sin \theta} = 2(46 \text{ ksi})(0.465 \text{ in.}) \left[ 5(0.698 \text{ in.}) + 6.00 \text{ in.} \right] \]
\[ = 406 \text{ kips} \]
\[ P_u = 406 \text{ kips} \]
The available strength of the branch is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi P_a = 1.00 \left( 406 \text{ kips} \right)$</td>
<td>$P_a = 406 \text{ kips}$</td>
</tr>
<tr>
<td>$= 406 \text{ kips}$</td>
<td>$\Omega = 1.50$</td>
</tr>
<tr>
<td>Branch utilization $= \frac{2.67 \text{ kips}}{406 \text{ kips}}$</td>
<td>$= 271 \text{ kips}$</td>
</tr>
<tr>
<td>$= 0.00658$</td>
<td>Branch utilization $= \frac{1.91 \text{ kips}}{271 \text{ kips}}$</td>
</tr>
<tr>
<td>$= 0.00705$</td>
<td></td>
</tr>
</tbody>
</table>

**Local crippling of chord sidewalls**

For branches in compression and with $\beta = 1$, the limit state of sidewall local crippling must also be checked. From AISC Specification Section K2.3b(c), the nominal strength of the branch for this limit state is determined from:

$$P_{\sin \theta} = 1.6r^2 \left[ 1 + \frac{3N}{H - 3f} \right] \sqrt{EF_f Q_f}$$

(Spec. Eq. K2-16 and Table 8-2)

where

$$Q_f = 1.3 - 0.4 \frac{U}{\beta} \leq 1.0 \quad \text{for chord (connecting surface) in compression (left of joint)}$$

(Spec. Eq. K2-10 and Table 8-2)

$$U = \frac{P_a + M_r}{AF_c + SF_c}$$

(Spec. Eq. K2-12 and Table 8-2)

From Chapter 2 of ASCE 7, the required strengths, $P_r$ and $M_r$, are determined as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_r = P_a$</td>
<td>$P_r = P_a$</td>
</tr>
<tr>
<td>$= 1.2(-2.87 \text{ kips}) + 1.6(-2.87 \text{ kips})$</td>
<td>$= -2.87 \text{ kips} - 2.87 \text{ kips}$</td>
</tr>
<tr>
<td>$= -8.04 \text{ kips}$ (tension)</td>
<td>$= -5.74 \text{ kips}$ (tension)</td>
</tr>
<tr>
<td>$M_r = M_a$</td>
<td>$M_r = M_a$</td>
</tr>
<tr>
<td>$= 1.2(82.5 \text{ kip-in.}) + 1.6(82.5 \text{ kip-in.})$</td>
<td>$= 82.5 \text{ kip-in.} + 82.5 \text{ kip-in.}$</td>
</tr>
<tr>
<td>$= 231 \text{ kip-in.}$ (causing compression)</td>
<td>$= 165 \text{ kip-in.}$ (causing compression)</td>
</tr>
<tr>
<td>$F_c = F_c$</td>
<td>$F_c = 0.6F_c$</td>
</tr>
<tr>
<td>$= 46 \text{ ksi}$</td>
<td>$= 27.6 \text{ ksi}$</td>
</tr>
<tr>
<td>$U = \frac{-8.04 \text{ kips}}{9.74 \text{ in.}^2(46 \text{ ksi})} + \frac{231 \text{ kip-in.}}{16.1 \text{ in.}^2(46 \text{ ksi})}$</td>
<td>$U = \frac{-5.74 \text{ kips}}{9.74 \text{ in.}^2(27.6 \text{ ksi})} + \frac{165 \text{ kip-in.}}{16.1 \text{ in.}^2(27.6 \text{ ksi})}$</td>
</tr>
<tr>
<td>$= 0.294$</td>
<td>$= 0.350$</td>
</tr>
</tbody>
</table>

$Q_f = 1.3 - 0.4 \left( \frac{0.294}{1.00} \right)$

Use $Q_f = 1.0$

$= 1.18 \geq 1.0$

$= 1.16 \geq 1.0$

Use $Q_f = 1.0$
The available strength of the branch for the limit state of sidewall local crippling is determined as follows:

\[
P_t = \frac{1.6 t^2 \left[ 1 + \frac{3N}{H - 3t} \right] \sqrt{E_F Q_f}}{\sin \theta}
\]

\[
= \frac{1.6 (0.465 \text{ in.})^2 \left[ 1 + \frac{3(6.00 \text{ in.})}{6.00 \text{ in.} - 3(0.465 \text{ in.})} \right] \sqrt{29,000 \text{ ksi}(46 \text{ ksi})(1.0)}}{\sin 90^\circ}
\]

\[= 1,961 \text{ kips}\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi P_n = 0.75(1,961 \text{ kips}))</td>
<td>(P_n = 1,961 \text{ kips})</td>
</tr>
<tr>
<td>(= 1,471 \text{ kips})</td>
<td>(\Omega = \frac{2.00}{2.00})</td>
</tr>
<tr>
<td>Branch utilization = (\frac{2.67 \text{ kips}}{1,471 \text{ kips}})</td>
<td>Branch utilization = (\frac{1.91 \text{ kips}}{981 \text{ kips}})</td>
</tr>
<tr>
<td>(= 0.00182)</td>
<td>(= 0.00195)</td>
</tr>
</tbody>
</table>

**Local yielding of branch due to uneven load distribution**

From AISC Specification Section K2.3b, the nominal strength of the branch for the limit state of local yielding due to uneven load distribution is:

\[
P_n = F_{\text{lt}} t_b \left(2H_b + 2b_{\text{out}} - 4t_b\right)
\]

(Spec. Eq. K2-18 and Table 8-2)

where

\[
b_{\text{out}} = \frac{10}{B/t} \left(\frac{F_{\text{lt}}}{F_{\text{lt}t_b}}\right) B_b \leq B_t
\]

(Spec. Eq. K2-19 and Table 8-2)

\[
= \frac{10}{12.9} \left(\frac{46 \text{ ksi}(0.465 \text{ in.})}{46 \text{ ksi}(0.291 \text{ in.})}\right) (6.00 \text{ in.})
\]

\[= 7.43 \text{ in.} \geq B_b = 6.00 \text{ in.}\]

therefore \(b_{\text{out}} = 6.00 \text{ in.}\).

The nominal strength is:

\[
P_n = 46 \text{ ksi}(0.291 \text{ in.}) \left[2(6.00 \text{ in.}) + 2(6.00 \text{ in.}) - 4(0.291 \text{ in.})\right]
\]

\[= 306 \text{ kips}\]

The available strength of the branch is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi P_n = 0.95(306 \text{ kips}))</td>
<td>(P_n = 306 \text{ kips})</td>
</tr>
<tr>
<td>(= 291 \text{ kips})</td>
<td>(\Omega = \frac{1.58}{1.58})</td>
</tr>
<tr>
<td>Branch utilization = (\frac{2.67 \text{ kips}}{291 \text{ kips}})</td>
<td>Branch utilization = (\frac{1.91 \text{ kips}}{194 \text{ kips}})</td>
</tr>
<tr>
<td>(= 0.00917)</td>
<td>(= 0.00985)</td>
</tr>
</tbody>
</table>

This limit state is the most critical for branch axial loading.
**Analysis under Branch Moment Loading**

**Sidewall local yielding**

From AISC Specification Section K3.3b(b), the limit state of local yielding of the chord sidewalls must be checked if $\beta \geq 0.85$. Because $\beta = B_b / B = 6.00\text{ in.}/6.00\text{ in.} = 1.0$, the nominal flexural strength of the branch for this limit state is determined from:

$$M_n = 0.5F_y^t(H_b + 5t)^2$$

(Spec. Eq. K3-12 and Table 9-2)

where

$$F_y^t = F_y = 46\text{ ksi}$$

Therefore

$$M_n = 0.5(46\text{ ksi})(0.465\text{ in.})[6.00\text{ in.} + 5(0.465\text{ in.})]^2 = 741\text{ kip-in.}$$

The available flexural strength of the branch for this limit state is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi M_n = 1.00(741\text{ kip-in.})$</td>
<td>$\frac{M_n}{\Omega} = \frac{741\text{ kip-in.}}{1.50} = 494\text{ kip-in.}$</td>
</tr>
<tr>
<td>$= 741\text{ kip-in.}$</td>
<td>Branch utilization $= \frac{330\text{ kip-in.}}{494\text{ kip-in.}} = 0.668$</td>
</tr>
<tr>
<td>Branch utilization $= \frac{462\text{ kip-in.}}{741\text{ kip-in.}} = 0.623$</td>
<td></td>
</tr>
</tbody>
</table>

**Local yielding of the branch due to uneven load distribution**

From AISC Specification Section K3.3b(c), the nominal flexural strength of the branch for the limit state of local yielding due to uneven load distribution is:

$$M_n = F_{yb} \left[ Z_b + \left(1 - \frac{b_{oc}}{B_b}\right) B_b H_b t_b \right]$$

(Spec. Eq. K3-13 and Table 9-2)

where $b_{oc} = 6.00\text{ in.}$ from the previous limit state check

Therefore

$$M_n = 46\text{ ksi} \left[13.6\text{ in.}^3 - \left(1 - \frac{6.00\text{ in.}}{6.00\text{ in.}}\right)(6.00\text{ in.})(6.00\text{ in.})(0.291\text{ in.})\right] = 626\text{ kip-in.}$$

The available flexural strength of the branch for this limit state is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi M_n = 0.95(626\text{ kip-in.})$</td>
<td>$M_n = \frac{626\text{ kip-in.}}{1.58} = 396\text{ kip-in.}$</td>
</tr>
<tr>
<td>$= 595\text{ kip-in.}$</td>
<td>Branch utilization $= \frac{330\text{ kip-in.}}{396\text{ kip-in.}} = 0.833$</td>
</tr>
<tr>
<td>Branch utilization $= \frac{462\text{ kip-in.}}{595\text{ kip-in.}} = 0.776$</td>
<td></td>
</tr>
</tbody>
</table>

This limit state is therefore the most critical for branch moment loading.
Total utilization of the connection capacity

Using LRFD:

\[
\frac{P_n}{\phi P_n} + \frac{M_{n-q}}{\phi M_{n-q}} \leq 1.0
\]

= 0.01(\text{axial}) + 0.78(\text{moment})

= 0.79 \leq 1.0 \quad \text{o.k.}

Using ASD:

\[
\frac{P_n}{(P_n/\Omega)} + \frac{M_{n-q}}{(M_{n-q}/\Omega)} \leq 1.0
\]

= 0.01(\text{axial}) + 0.83(\text{moment})

= 0.84 \leq 1.0 \quad \text{o.k.}

The summations differ between LRFD and ASD because of the particular live load to dead load ratio in this example.

Example 9.3—Cross-Connection with Rectangular HSS (Out-of-Plane Bending)

Given:

Verify the adequacy of the welded cross-connection shown here, where the branches are subject to balanced out-of-plane bending moments. Loads consist of 25% dead load and 75% live load and are shown in Figure 9-4. Assume the welds are strong enough to develop the yield strength of the connected branch wall at all locations around the branch.

From AISC Manual Table 2-3, the material properties are as follows:

All members
ASTM A500 Grade B
\(F_y = 46\) ksi
\(F_u = 58\) ksi

\[
\begin{align*}
\text{HSS8} \times 6 \times \% \\
\text{ASTM A500 Gr. C} \\
\text{HSS16} \times 12 \times \frac{1}{2}\% \\
\text{ASTM A500 Gr. C} \\
P_L = 300\ kips \\
P_D = 100\ kips
\end{align*}
\]

\[
\begin{align*}
\text{ML} = 105\ \text{kip-in.} \\
M_D = 35.0\ \text{kip-in.}
\end{align*}
\]

\[
\begin{align*}
\text{HSS8} \times 6 \times \% \\
\text{ASTM A500 Gr. C} \\
\text{ML} = 105\ \text{kip-in.} \\
M_D = 35.0\ \text{kip-in.}
\end{align*}
\]

Fig. 9-4. Cross-connection with rectangular HSS.
From AISC Manual Table 1-11, the HSS geometric properties are as follows:

HSS16×12×⅜

\[ A = 30.3 \text{ in.}^2 \]
\[ B = 16.0 \text{ in.} \]
\[ H = 12.0 \text{ in.} \]
\[ t = 0.581 \text{ in.} \]

HSS8×6×⅜

\[ A_b = 8.97 \text{ in.}^2 \]
\[ B_b = 6.00 \text{ in.} \]
\[ H_b = 8.00 \text{ in.} \]
\[ t_b = 0.349 \text{ in.} \]

Solution:

Limits of applicability

From AISC Specification Section K3.3a and Table 9-2A, the limits of applicability for this connection are:

\[ \theta = 90^\circ \equiv 90^\circ \quad \text{o.k.} \]
\[ \frac{H}{t} = \frac{12.0}{0.581} = 20.7 \leq 35 \quad \text{o.k.} \]
\[ \frac{B}{t} = \frac{16.0}{0.581} = 27.5 \leq 35 \quad \text{O.K.} \]
\[ \frac{B_b}{t_b} = \frac{6.00}{0.349} = 17.2 \leq 35 \quad \text{O.K.} \]
\[ \frac{H_b}{t_b} = \frac{8.00}{0.349} = 22.9 \leq 35 \quad \text{O.K.} \]
\[ \frac{B_b}{B} = \frac{6.00}{16.0} = 0.375 \geq 0.25 \quad \text{O.K.} \]
Required moment capacity (expressed as a moment in the branch)

From Chapter 2 of ASCE 7, the required strength of the connection due to the moments in the branches is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_u = 1.2(35.0 \text{ kip-in.}) + 1.6(105 \text{ kip-in.})$</td>
<td>$M_u = 35.0 \text{ kip-in.} + 105 \text{ kip-in.}$</td>
</tr>
<tr>
<td>= 210 kip-in.</td>
<td>= 140 kip-in.</td>
</tr>
</tbody>
</table>

Chord wall plastification

According to AISC Specification Section K3.3c(a), the limit state of chord wall plastification need not be checked when $\beta > 0.85$. Because $\beta = B/B_b = 0.375 \leq 0.85$, this limit state must be considered. From AISC Specification Section K3.3c(a), the nominal flexural strength of the main member chord is:

$$M_u = F_f t_r \left[ \frac{0.5B_b(1+\beta)}{(1-\beta)} + \sqrt{ \frac{2BB_b(1+\beta)}{(1-\beta)} } \right] Q_f$$

(Spec. Eq. K3-15 and Table 9-2)

where

$$Q_f = 1.3 - 0.4 \frac{U}{\beta} \leq 1.0 \quad \text{for chord (connecting surface) in compression}$$  

(Spec. Eq. K3-9 and Table 9-2)

$$U = \left[ \frac{P}{AF} + \frac{M}{SF} \right]$$

(Spec. Eq. K3-10 and Table 9-2)
From Chapter 2 of ASCE 7, the required strength, $P_r$, is:

$$\begin{align*}
LRFD & \\
\frac{P_r}{P_a} &= 1.2 \left(100 \text{ kips} \right) + 1.6 \left(300 \text{ kips} \right) \\
&= 600 \text{ kips} \\
M_r &= 0 \\
F_c &= F_y \\
U &= \frac{600 \text{ kips}}{30.3 \text{ in.}^2 (46 \text{ ksi})} \\
&= 0.430 \\
Q_f &= 1.3 - 0.4 \left(\frac{0.430}{0.375}\right) \\
&= 0.841
\end{align*}$$

$$\begin{align*}
ASD & \\
\frac{P_r}{P_a} &= 100 \text{ kips} + 300 \text{ kips} \\
&= 400 \text{ kips} \\
M_r &= 0 \\
F_c &= 0.6 F_y \\
U &= \frac{400 \text{ kips}}{30.3 \text{ in.}^2 (27.6 \text{ ksi})} \\
&= 0.478 \\
Q_f &= 1.3 - 0.4 \left(\frac{0.478}{0.375}\right) \\
&= 0.790
\end{align*}$$

The available flexural strength of the main (chord) member for the limit state of chord wall plastification is determined as follows:

$$\begin{align*}
LRFD & \\
M_n &= 46 \text{ ksi} \left(0.581 \text{ in.}^2\right)^2 \\
& \times \left[\frac{0.5 \left(8.00 \text{ in.}\right) \left(1 + 0.375\right)}{\left(1 - 0.375\right)} \right] \\
& \left[\frac{2 \left(16.0 \text{ in.}\right) \left(6.00 \text{ in.}\right) \left(1 + 0.375\right)}{\left(1 - 0.375\right)} \right] \\
&= 383 \text{ kip-in.} \\
\phi M_n &= 1.00 \left(383 \text{ kip-in.}\right) \\
&= 383 \text{ kip-in.} \\
383 \text{ kip-in.} > 210 \text{ kip-in.} & \text{ o.k.}
\end{align*}$$

$$\begin{align*}
ASD & \\
M_n &= 46 \text{ ksi} \left(0.581 \text{ in.}^2\right)^2 \\
& \times \left[\frac{0.5 \left(8.00 \text{ in.}\right) \left(1 + 0.375\right)}{\left(1 - 0.375\right)} \right] \\
& \left[\frac{2 \left(16.0 \text{ in.}\right) \left(6.00 \text{ in.}\right) \left(1 + 0.375\right)}{\left(1 - 0.375\right)} \right] (0.790) \\
&= 360 \text{ kip-in.} \\
\Omega &= \frac{360 \text{ kip-in.}}{240 \text{ kip-in.}} \\
&= 1.50 \\
240 \text{ kip-in.} > 140 \text{ kip-in.} & \text{ o.k.}
\end{align*}$$

The limit states that are applicable for $\beta > 0.85$ do not control and need not be checked because $\beta = 0.375$. The limit state of chord distortional failure is not applicable as this cross-connection has self-balancing branch moments.
Symbols

\( A \quad = \quad \) chord member cross-sectional area, in.\(^2\) (mm\(^2\)); refers to main member for HSS-to-HSS connection

\( A_b \quad = \quad \) branch member cross-sectional area, in.\(^2\) (mm\(^2\)); nominal unthreaded body area of bolt, in.\(^2\) (mm\(^2\))

\( A_n \quad = \quad \) effective net area, in.\(^2\) (mm\(^2\))

\( A_{gs} \quad = \quad \) gross cross-sectional area of member, in.\(^2\) (mm\(^2\))

\( A_{as} \quad = \quad \) gross area subject to shear, in.\(^2\) (mm\(^2\))

\( A_{ns} \quad = \quad \) net cross-sectional area of member, in.\(^2\) (mm\(^2\))

\( A_{w} \quad = \quad \) net area subject to tension, in.\(^2\) (mm\(^2\))

\( A_{wb} \quad = \quad \) projected bearing area, in.\(^2\) (mm\(^2\))

\( A_{e} \quad = \quad \) effective area of the weld, in.\(^2\) (mm\(^2\))

\( a \quad = \quad \) \( e/L \); distance from bolt centerline to edge of end plate, in. (mm)

\( a' \quad = \quad \) \( a + d_b/2 \)

\( a_e \quad = \quad a \leq 1.25b \) = effective value of \( a \), for use in bolt-prying calculations

\( B \quad = \quad \) overall width of rectangular HSS main member, measured 90° to the plane of the connection, in. (mm)

\( B_b \quad = \quad \) overall width of rectangular HSS branch member, measured 90° to the plane of the connection, in. (mm)

\( B_{be} \quad = \quad \) overall width of rectangular HSS overlapping branch member, in. (mm)

\( B_{bij} \quad = \quad \) overall width of rectangular HSS overlapped branch member, in. (mm)

\( B_{e} \quad = \quad \) effective width of plate, measured 90° to the plane of the connection, in. (mm)

\( B_p \quad = \quad \) width of plate, measured 90° to the plane of the connection, in. (mm)

\( b \quad = \quad \) distance from bolt centerline to face of connected member, in. (mm)

\( b' \quad = \quad \) \( b - d_b/2 \) = distance from face of bolt to face of connected member in. (mm)

\( b_{ec} \quad = \quad \) effective width of the branch face welded to the chord, in. (mm)

\( b_{ec} \quad = \quad \) effective width of the branch face welded to the overlapped brace, in. (mm)

\( b_f \quad = \quad \) flange width, in. (mm)

\( C \quad = \quad \) coefficient for eccentrically loaded weld groups; compression force, kips (N)

\( C_1 \quad = \quad \) electrode strength coefficient

\( C_a \quad = \quad \) Compressive force using ASD load combinations, kips (N)

\( C_r \quad = \quad \) Compressive force using LRFD or ASD load combinations, kips (N)

\( C_u \quad = \quad \) Compressive force using LRFD load combinations, kips (N)

\( D \quad = \quad \) outside diameter of round HSS main member, in. (mm); number of sixteenths-of-an-inch in fillet weld size

\( D_b \quad = \quad \) outside diameter of round HSS branch member, in. (mm)

\( D_{eff} \quad = \quad \) effective weld size, sixteenths-of-an-inch

\( D_{min} \quad = \quad \) minimum number of sixteenths-of-an-inch in the fillet weld size

\( d \quad = \quad \) nominal fastener diameter, in. (mm); full nominal depth of the section, in. (mm)

\( d_b \quad = \quad \) bolt diameter, in. (mm)

\( d_{ho} \quad = \quad \) bolt hole diameter, in. (mm)

\( d_w \quad = \quad \) diameter of part in contact with the inner surface of the HSS, or diameter of welded stud, in. (mm)

\( E \quad = \quad \) modulus of elasticity of steel = 29,000 ksi (200,000 MPa); effective throat thickness of weld, in. (mm)

\( e \quad = \quad \) eccentricity in a truss connection, positive being away from the branches, in. (mm); eccentricity of the required force with respect to the centroid of the bolt group or weld group, in. (mm); eccentricity of the required load with respect to the center of the connected plates, in. (mm)

\( F_s \quad = \quad \) available stress = \( F_y \) (LRFD) or 0.6\( F_y \) (ASD), ksi (MPa)

\( F_{cr} \quad = \quad \) available critical buckling stress, ksi (MPa)

\( F_{RM} \quad = \quad \) nominal strength of the base metal, ksi (MPa)

\( F_{XXX} \quad = \quad \) electrode classification number, ksi (MPa)

\( F_f \quad = \quad \) force in the flange, kips (N)

\( F_{nt} \quad = \quad \) nominal tensile stress, ksi (MPa)
\[ F_u = \text{specified minimum ultimate strength of material, ksi (MPa); refers to main member for HSS-to-HSS connections; specified minimum tensile strength of connecting element, ksi (MPa)} \]

\[ F_{ub} = \text{specified minimum tensile strength of HSS branch member material, ksi (MPa)} \]

\[ F_{wp} = \text{specified minimum ultimate strength of plate, ksi (MPa)} \]

\[ F_u = \text{nominal strength of weld metal, ksi (MPa)} \]

\[ F_{yw} = \text{specified minimum yield stress of web, ksi (MPa)} \]

\[ g = \text{gap between toes of branch members in a gapped K-connection, neglecting the welds, in. (mm); transverse center-to-center spacing (gage) between fastener gage lines, in. (mm)} \]

\[ H = \text{overall height of rectangular HSS main member, measured in the plane of the connection, in. (mm)} \]

\[ H_b = \text{overall height of rectangular HSS branch member, measured in the plane of the connection, in. (mm)} \]

\[ I = \text{moment of inertia, in.}^4 (\text{mm}^4) \]

\[ K = \text{effective length factor} \]

\[ k = \text{outside corner radius of the HSS, which is permitted to be taken as 1.5}t \text{ if unknown, in. (mm); distance from outer face of flange to the web toe of fillet, in. (mm) } \]

\[ k_i = \text{distance from web centerline to flange toe of fillet, in. (mm)} \]

\[ L = \text{length of fastener or weld, in. (mm)} \]

\[ L_e = \text{length of connection, in. (mm); clear distance, in the direction of the force, between the edge of the hole and the edge of the adjacent hole or edge of the material, in. (mm)} \]

\[ L_{ew} = \text{total effective weld length of groove and fillet welds to rectangular HSS, in. (mm)} \]

\[ L_{gy} = \text{gross length subject to shear, in. (mm)} \]

\[ l = \text{weld length, in. (mm); length of connection, in. (mm)} \]

\[ M = \text{bending moment, kip-in. (N-mm)} \]

\[ M_u = \text{required flexural strength (ASD), kip-in. (N-mm)} \]

\[ M_e = \text{available flexural strength, kip-in. (N-mm)} \]

\[ M_D = \text{specified dead load bending moment, kip-in. (N-mm)} \]

\[ M_L = \text{specified live load bending moment, kip-in. (N-mm)} \]

\[ M_n = \text{nominal moment capacity, kip-in. (N-mm)} \]

\[ M_{ps} = \text{nominal plastic bending moment capacity about x-axis, kip-in. (N-mm)} \]

\[ M_r = \text{required flexural strength, kip-in. (N-mm)} \]

\[ M_{rip} = \text{required in-plane flexural strength in branch, using LRFD or ASD load combinations as applicable, kip-in. (N-mm)} \]

\[ M_{rip} = \text{required out-of-plane flexural strength in branch, using LRFD or ASD load combinations as applicable, kip-in. (N-mm)} \]

\[ M_r = \text{required flexural strength (LRFD), kip-in. (N-mm)} \]

\[ N = \text{bearing length of the load, measured parallel to the axis of the HSS member (or measured across the width of the HSS in the case of loaded cap plates), in. (mm)} \]

\[ n = \text{number of bolts} \]

\[ Q_f = \text{reduction factor (≤1.0) to account for the effect of normal stresses in the chord member connecting surface} \]

\[ Q_g = \text{factor to account for gap or overlap in round HSS connections} \]

\[ q = \text{the overlap length measured along the connecting face of the chord beneath the two branches, in. (mm)} \]
\( R \) = radius of joint surface, in. (mm)
\( R_a \) = required strength (ASD), kips (N)
\( R_c \) = available tensile strength of one bolt, kips (N)
\( R_n \) = nominal strength, kips (N)
\( R_r \) = required strength, kips (N)
\( R_u \) = required strength (LRFD), kips (N)
\( r \) = radius of gyration, in. (mm)
\( r_n \) = nominal strength per bolt, kips (N)
\( S \) = chord elastic section modulus, in.\(^3\) (mm\(^3\)); partial-joint-penetration groove weld depth, in. (mm)
\( s \) = bolt spacing, in. (mm)
\( t \) = design wall thickness of HSS main member, in. (mm); thickness of element, in. (mm)
\( T_a \) = tensile force using ASD load combinations, kips (N)
\( t_b \) = design wall thickness of HSS branch member, in. (mm)
\( T_e \) = available tensile strength, kips (N)
\( t_f \) = thickness of flange, in. (mm)
\( t_{min} \) = minimum thickness of HSS material to develop full strength of the weld, in. (mm); minimum thickness required to eliminate prying action, in. (mm)
\( t_p \) = thickness of plate, in. (mm)
\( T_r \) = tensile force using LRFD or ASD load combinations, kips (N)
\( t_s \) = thickness of stem plate, in. (mm)
\( T_u \) = tensile force using LRFD load combinations, kips (N)
\( t_w \) = thickness of web, in. (mm)
\( U \) = chord utilization ratio; shear lag factor
\( U_{hs} \) = reduction coefficient, used in calculating block shear rupture strength
\( V_D \) = specified shear dead load, kips (N)
\( V_h \) = horizontal component of applied load, kips (N)
\( V_L \) = specified shear live load, kips (N)
\( V_r \) = required shear strength, kips (N)
\( V_v \) = vertical component of applied load, kips (N)
\( W \) = width across flats of nut, in. (mm); plate width, in. (mm)
\( W_{eff} \) = effective weld size, in. (mm)
\( W_{eq} \) = equivalent weld size, in. (mm)
\( \bar{\alpha} \) = connection eccentricity, in. (mm)
\( Z \) = plastic section modulus about the axis of bending, in.\(^3\) (mm\(^3\))
\( Z_b \) = branch plastic section modulus about the axis of bending, in.\(^3\) (mm\(^3\))
\( \beta \) = width ratio; the ratio of branch diameter to chord diameter = \( D_b/D \) for round HSS; the ratio of overall branch width to chord width = \( B_b/B \) for rectangular HSS
\( \beta_{eff} \) = effective width ratio; the sum of the perimeters of the two branch members in a K-connection divided by eight times the chord width
\( \beta_{op} \) = effective outside punching parameter = \( 5\beta\gamma \), but \( \leq \beta \)
\( \delta \) = \( 1 – d_s/p \)
\( \gamma \) = chord slenderness ratio; the ratio of one-half the diameter to the wall thickness = \( D/2t \) for round HSS; the ratio of one-half the width to wall thickness = \( B/2t \) for rectangular HSS
\( \zeta \) = gap ratio; the ratio of the gap between the branches of a gapped K-connection to the width of the chord = \( g/B \) for rectangular HSS
\( \eta \) = load length parameter, applicable only to rectangular HSS; the ratio of the length of contact of the branch with the chord in the plane of the connection to the chord width = \( N/B \), where \( N = H_b/\sin\theta \)
\( \phi \) = resistance factor
\( \phi_{BM} \) = resistance factor for base metal
\( \phi_b \) = resistance factor for flexure
\( \phi_r \) = resistance factor for shear
\( \phi_{w} \) = resistance factor for weld metal
\( \Omega \) = safety factor
\( \Omega_{BM} \) = safety factor for base metal
\( \Omega_b \) = safety factor for flexure
\( \Omega_r \) = safety factor for shear
\( \Omega_{w} \) = safety factor for weld metal
\( \rho \) = \( b'la' \)
\( \theta \) = acute angle between the branch and chord, degrees
References


AISC (1997), Hollow Structural Sections Connections Manual, American Institute of Steel Construction, Chicago, IL.

AISC (1999), Load and Resistance Factor Design Specification for Structural Steel Buildings, American Institute of Steel Construction, Chicago, IL.

AISC (2000), Load and Resistance Factor Design Specification for Steel Hollow Structural Sections, American Institute of Steel Construction, Chicago, IL.

AISC (2005a), Specification for Structural Steel Buildings, ANSI/AISC 360-05, American Institute of Steel Construction, Chicago, IL.


AISC (2005c), CD Companion, V.13.0, American Institute of Steel Construction, Chicago, IL.

AISI (2007), North American Specification for the Design of Cold-Formed Steel Structural Members, American Iron and Steel Institute, Washington, DC.


AWS (2008), Structural Welding Code—Steel, AWS D1.1-08, American Welding Society, Miami, FL.


CSA (2004), CAN/CSA G40.20-04/G40.21-04 General Requirements for Rolled or Welded Structural Quality Steel, Canadian Standards Association, Toronto, Canada.


